

Jet fragmentation functions in proton and heavy ion collisions

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BNL, 04/13/16



Outline

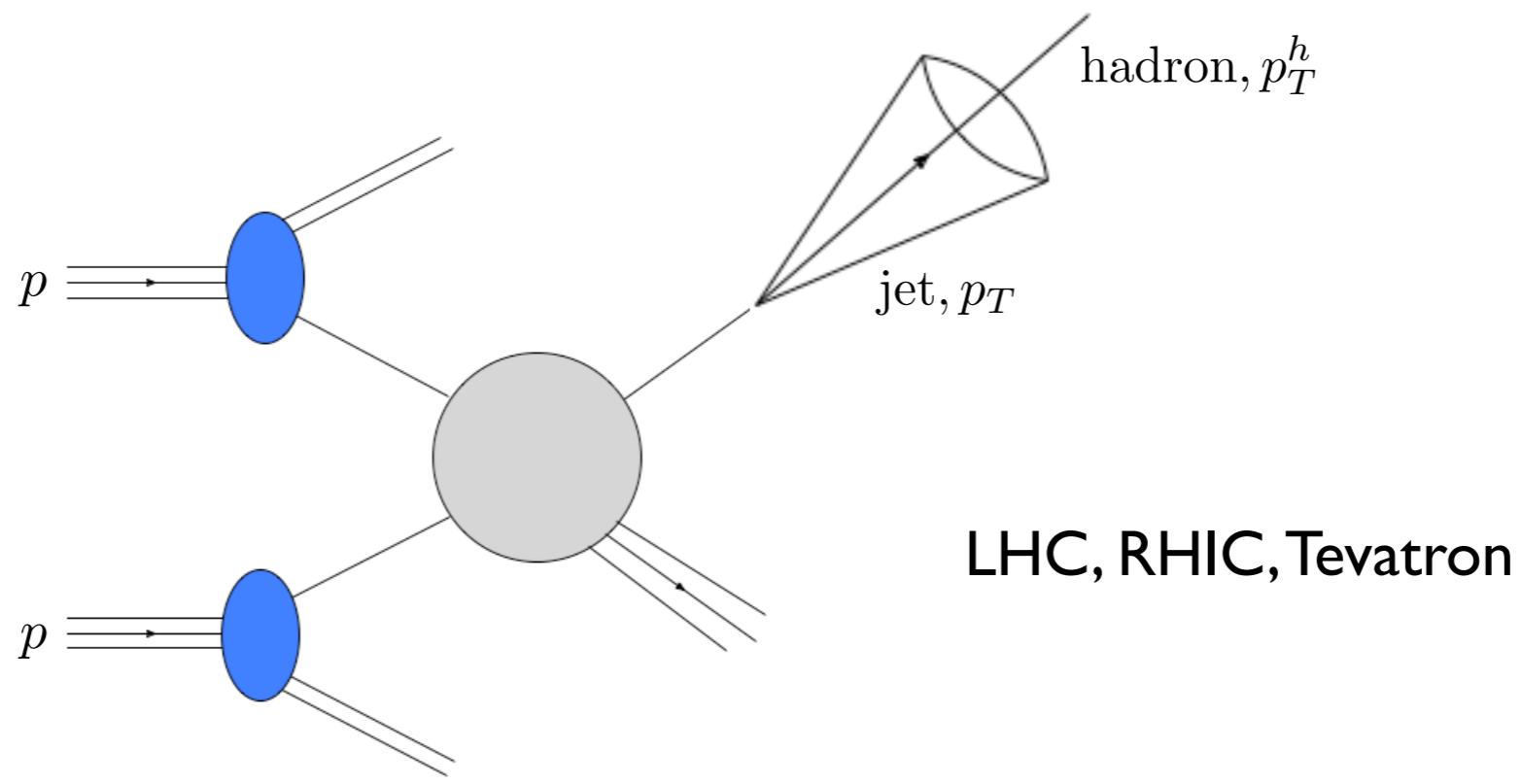
- Jet Fragmentation functions in pp collisions
Chien, Kang, FR, Vitev, Xing - '15
- Modification in AA
Chien, Kang, FR, Vitev, Xing - in preparation
- Conclusions

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Jet fragmentation function

- Jet substructure observable studying the distribution of hadrons inside a jet
- Probes jet dynamics at a more differential level
- Provides further constraints for fits of fragmentation functions
- Possible studies include spin correlations and
- the modification in heavy ion collisions



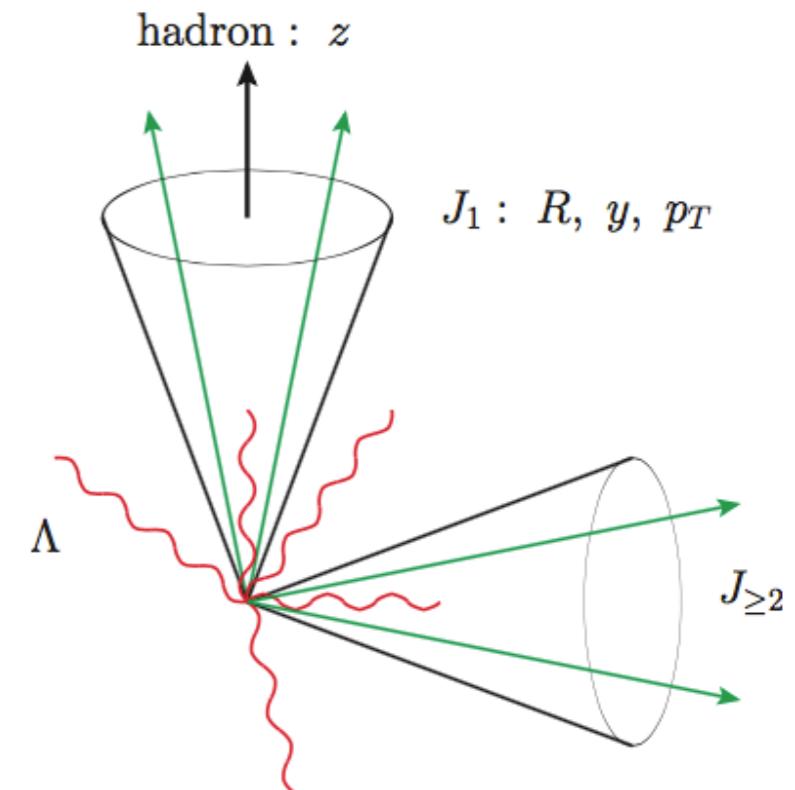
Jet fragmentation function

Definition:

$$F(z, p_T) = \frac{d\sigma^h}{dydp_Tdz} / \frac{d\sigma}{dydp_T}$$

where

$$z \equiv p_T^h / p_T$$



It describes the longitudinal momentum distribution
of hadrons inside a reconstructed jet

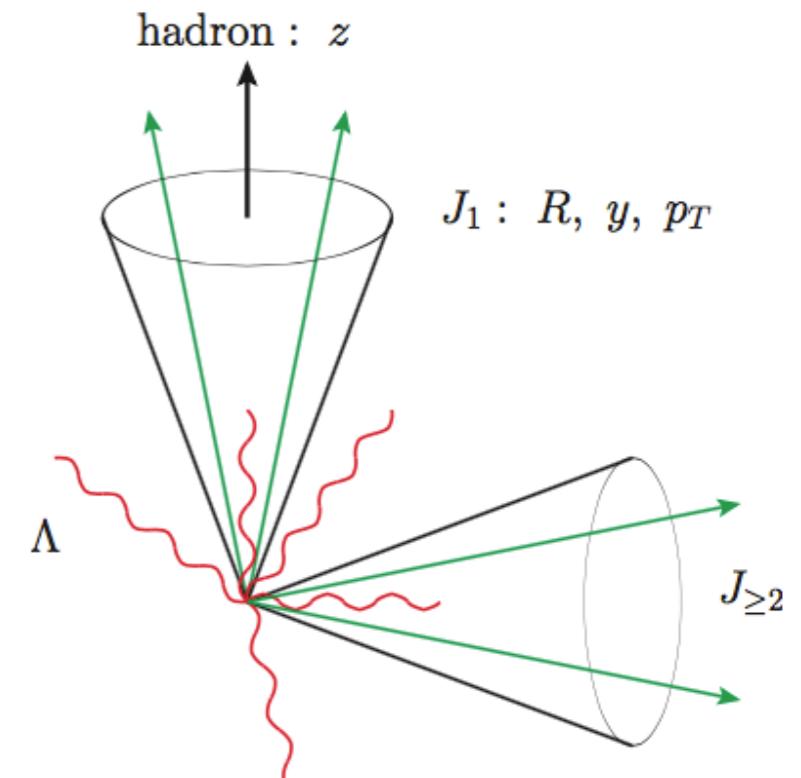
Jet fragmentation function in pp

- Fragmenting jet function studies within SCET

Procura, Stewart '10; Liu '11; Jain, Procura, Waalewijn '11 and '12; Procura, Waalewijn '12; Bauer, Mereghetti '14; Baumgart, Leibovich, Mehen, Rothstein '14, Bain, Dai, Hornig, Leibovich, Makris, Mehen '16 ...

- Jet fragmentation function studies at NLO for pp

*Arleo, Fontannaz, Guillet, Nguyen '14,
Kaufmann, Mukherjee, Vogelsang '15*



Jet fragmentation function in pp

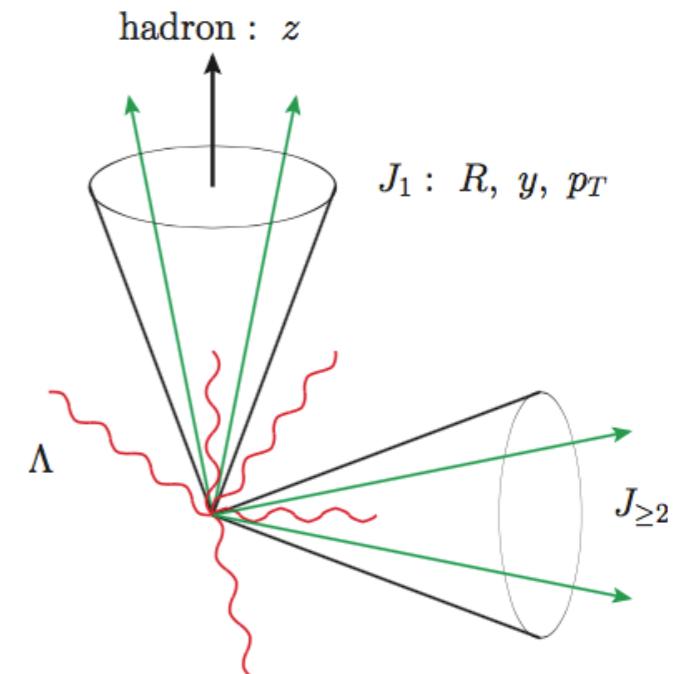
Factorization

$$\frac{\frac{d\sigma^h}{dy_i dp_{T_i} dz}}{\frac{d\sigma}{dy_i dp_{T_i}}} = \frac{H(y_i, p_{T_i}, \mu) \mathcal{G}_{\omega_1}^h(z, \mu) J_{\omega_2}(\mu) \cdots J_{\omega_N}(\mu) S_{n_1 n_2 \dots n_N}(\Lambda, \mu)}{H(y_i, p_{T_i}, \mu) J_{\omega_1}(\mu) \cdots J_{\omega_N}(\mu) S_{n_1 n_2 \dots n_N}(\Lambda, \mu)} = \frac{\mathcal{G}_{\omega_1}^h(z, \mu)}{J_{\omega_1}(\mu)}$$

Li, Li, Yuan '13
Chien, Vitev '15

$$\omega_i = 2p_{T_i}$$

→ $F(z, p_T) = \frac{1}{\sigma_{\text{tot}}} \sum_{i=q,g} \int_{\text{PS}} dy dp'_T \frac{d\sigma^i}{dy dp'_T} \frac{\mathcal{G}_i^h(\omega, R, z, \mu)}{J^i(\omega, R, \mu)}$



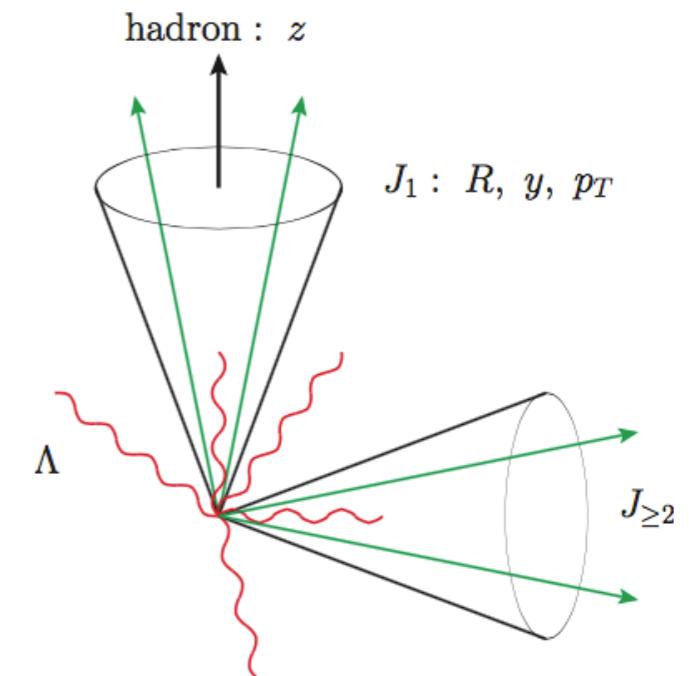
Fragmenting jet function

$$\mathcal{G}_{i,\text{bare}}^j(\omega, R, z, \mu) = \int \frac{dk_\perp^2}{(k_\perp^2)^{1+\epsilon}} \frac{\alpha_s}{2\pi} \frac{(e^{\gamma_E} \mu^2)^\epsilon}{\Gamma(1-\epsilon)} P_{ji}(z, \epsilon) \delta_{\text{alg}}$$

- where:

$$\delta_{\text{anti}-k_T} = \theta(k_\perp^2) \theta(z^2(1-z)^2 \omega^2 \tan^2(R/2) - k_\perp^2)$$

e.g. $P_{qq}(z, \epsilon) = C_F \left[\frac{1+z^2}{1-z} - \epsilon(1-z) \right]$



$\overline{\text{MS}}^*, \text{pure dim reg}, \mathcal{O}(\alpha_s)$

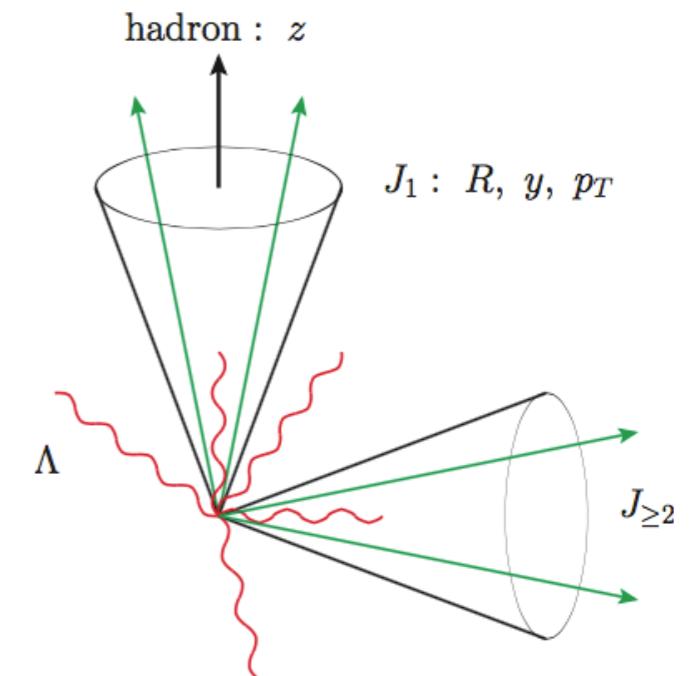
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$\overline{\text{MS}}, \text{pure dim reg}, \mathcal{O}(\alpha_s)$

- matching onto standard FFs $D_j^h(z, \mu)$ for $\mu_G \gg \Lambda_{\text{QCD}}$

$$\mathcal{G}_i^h(\omega, R, z, \mu) = \sum_j \int_z^1 \frac{dx}{x} \mathcal{J}_{ij}(\omega, R, x, \mu) D_j^h\left(\frac{z}{x}, \mu\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{\omega^2 \tan^2(R/2)}\right)$$

Matching coefficients

standard collinear fragmentation functions

Fragmenting jet function

Quark FJF at 1-loop:



$$\begin{aligned} \mathcal{J}_{qq}(\omega, R, z, \mu) = & \delta(1-z) + \frac{\alpha_s}{\pi} C_F \left[\delta(1-z) \left(\ln^2 \left(\frac{\omega \tan(R/2)}{\mu} \right) - \frac{\pi^2}{24} \right) + \hat{P}_{qq} \ln \left(\frac{\omega \tan(R/2)}{\mu} \right) \right. \\ & \left. + \hat{P}_{qq}(z) \ln z + (1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ \right] \end{aligned}$$

*Jain, Procura, Waalewijn '11,
Waalewijn '12*

where

$$\hat{P}_{qq}(z) = \frac{1+z^2}{(1-z)_+}$$

anti- k_T , $\overline{\text{MS}}$ scheme

Fragmenting jet function

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*Jain, Procura, Waalewijn '11,
Waalewijn '12*

$$J_q(\omega, R, \mu) = 1 + \frac{\alpha_s}{\pi} C_F \left[\ln^2 \left(\frac{\omega \tan(R/2)}{\mu} \right) - \frac{3}{2} \ln \left(\frac{\omega \tan(R/2)}{\mu} \right) + \frac{13}{4} - \frac{3\pi^2}{8} \right]$$

Ellis, Vermilion, Walsh, Hornig, Lee '10

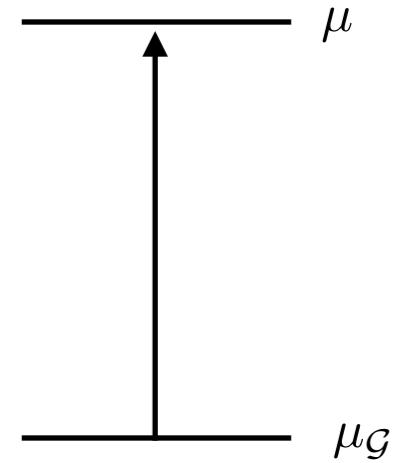
where

$$\hat{P}_{qq}(z) = \frac{1+z^2}{(1-z)_+}$$

anti- k_T , $\overline{\text{MS}}$ scheme

Resummation

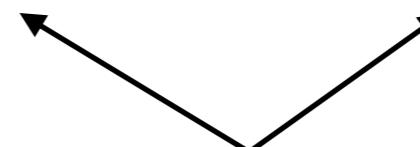
- solve RG equation $\mu \frac{d}{d\mu} \mathcal{G}_i^h(\omega, R, z, \mu) = \gamma_{\mathcal{G}}^i(\mu) \mathcal{G}_i^h(\omega, R, z, \mu)$



$$\mathcal{G}_i^h(\omega, R, z, \mu) = \mathcal{G}_i^h(\omega, R, z, \mu_G) \exp \left[\int_{\mu_G}^{\mu} \frac{d\mu'}{\mu'} \gamma_{\mathcal{G}}^i(\mu') \right]$$

where

$$\gamma_{\mathcal{G}}^i(\mu) = \Gamma_{\text{cusp}}^i(\alpha_s) \ln \frac{\mu^2}{\omega^2 \tan^2(R/2)} + \gamma^i(\alpha_s)$$



anomalous dimensions:

$$\Gamma_{\text{cusp}}^i = \sum_n \Gamma_{n-1}^i \left(\frac{\alpha_s}{4\pi} \right)^n$$

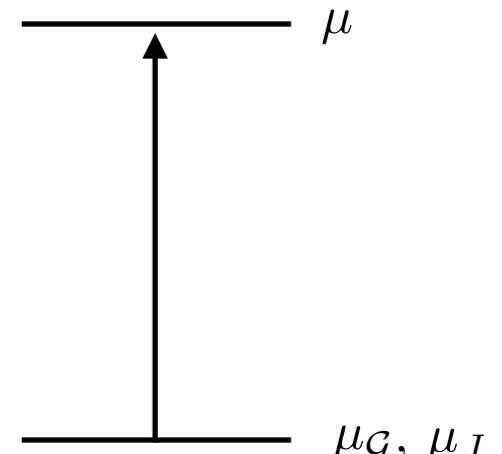
$$\gamma^i = \sum_n \gamma_{n-1}^i \left(\frac{\alpha_s}{4\pi} \right)^n$$

→ Resummation of $\ln R$

Resummation

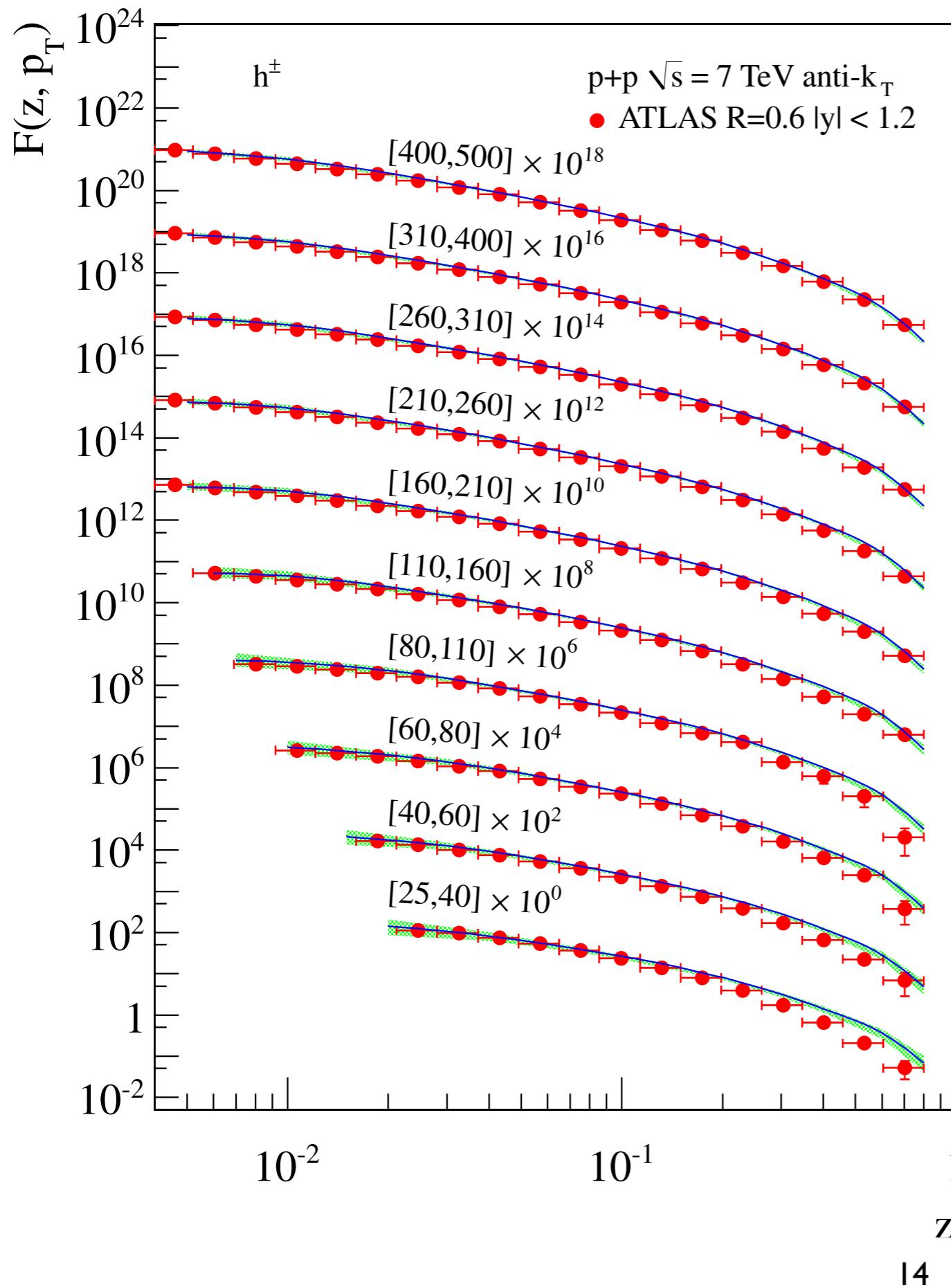
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$$\mathcal{G}_i^h(\omega, R, z, \mu) = \mathcal{G}_i^h(\omega, R, z, \mu_{\mathcal{G}}) \exp \left[\int_{\mu_{\mathcal{G}}}^{\mu} \frac{d\mu'}{\mu'} \gamma_{\mathcal{G}}^i(\mu') \right]$$



- same for the unmeasured jet function $J_{\omega_1}(\mu)$

→ $F_{\omega_1}(z, p_{T_i}) = \frac{\mathcal{G}_{\omega_1}^h(z, \mu)}{J_{\omega_1}(\mu)}$ is RG-invariant for $\mu_{\mathcal{G}} = \mu_J = p_{TR} = \omega \tan(R/2)$



Comparison to ATLAS data
at $\sqrt{s} = 7 \text{ TeV}$

Light charged hadrons $h = \pi + K + p$

QCD scale uncertainty

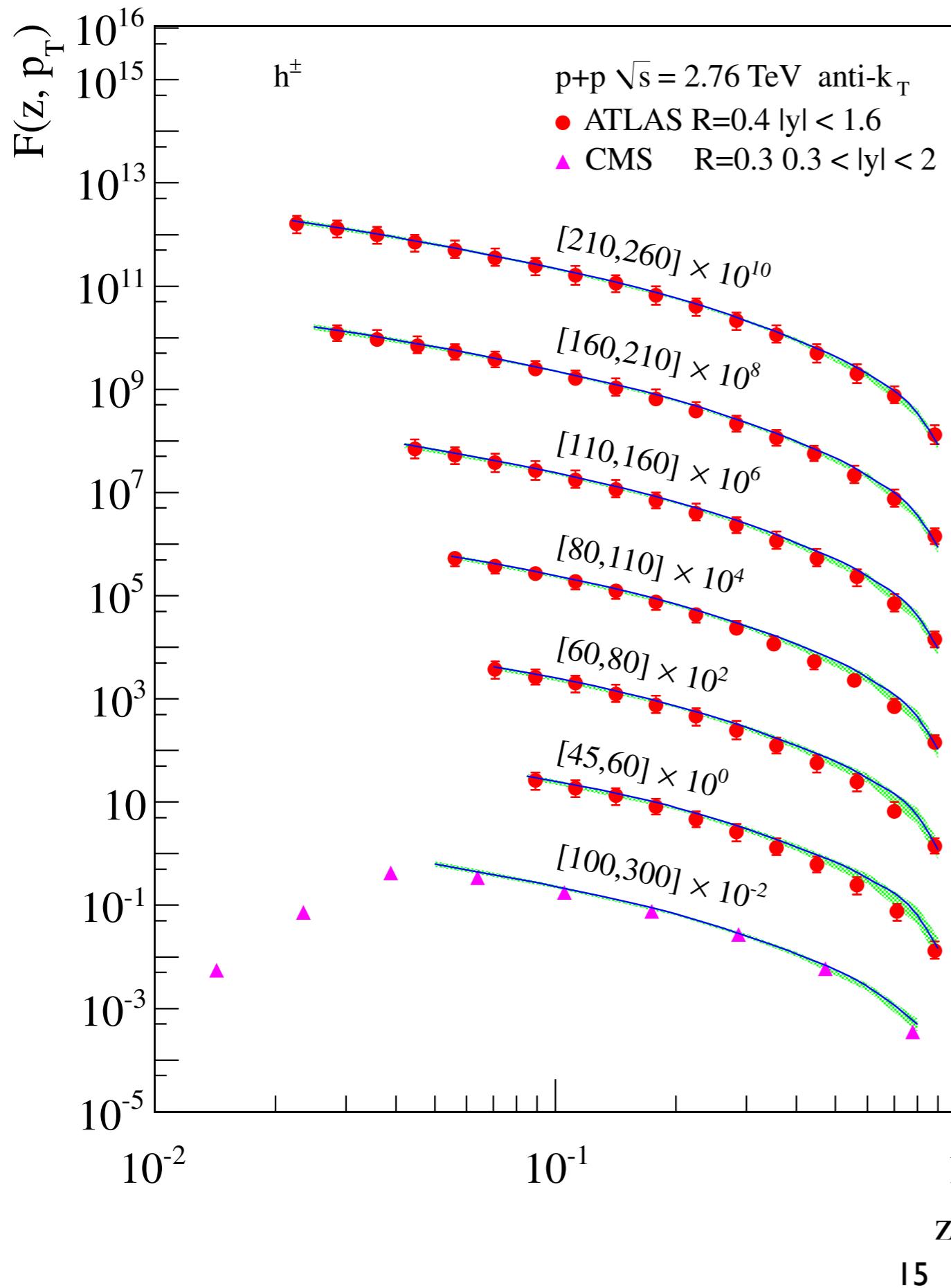
$$\mu \in [p_T/2, 2p_T],$$

$$\mu_J \in [p_{TR}/2, 2p_{TR}],$$

$$\mu_G \in [p_{TR}/2, 2p_{TR}]$$

$$p_{TR} = 2p_T \tan(R/2)$$

Using DSS FFs
de Florian, Sassot, Stratmann - '07



Comparison to ATLAS and CMS
data at $\sqrt{s} = 2.76 \text{ TeV}$

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QCD scale uncertainty

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Using DSS FFs
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Resummation of large logarithms

Convolution structure:

$$\mathcal{G}_i^h(\omega, R, z, \mu) = \sum_j \int_z^1 \frac{dx}{x} \mathcal{J}_{ij}(\omega, R, x, \mu) D_j^h\left(\frac{z}{x}, \mu\right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{\omega^2 \tan^2(R/2)}\right)$$



Threshold logarithms $\left(\frac{\ln(1-x)}{1-x}\right)_+$ become large at the partonic threshold $x \rightarrow 1$

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Threshold logarithms $\left(\frac{\ln(1-x)}{1-x}\right)_+$ become large at the partonic threshold $x \rightarrow 1$

→ $\mathcal{G}_q^h(\omega, R, z, \mu) = \left\{ 1 + \frac{\alpha_s}{\pi} C_F \left[\ln^2 \left(\frac{\omega \tan(R/2)(1-z)}{\mu} \right) - \frac{\pi^2}{24} \right] \right\} D_q^h(z, \mu) + \dots$

Choosing $\mu = \omega \tan(R/2)(1-z)$ resums logarithms of R and $(1-z)$

Both $\ln R$ and $\ln(1 - z)$
are simultaneously resummed

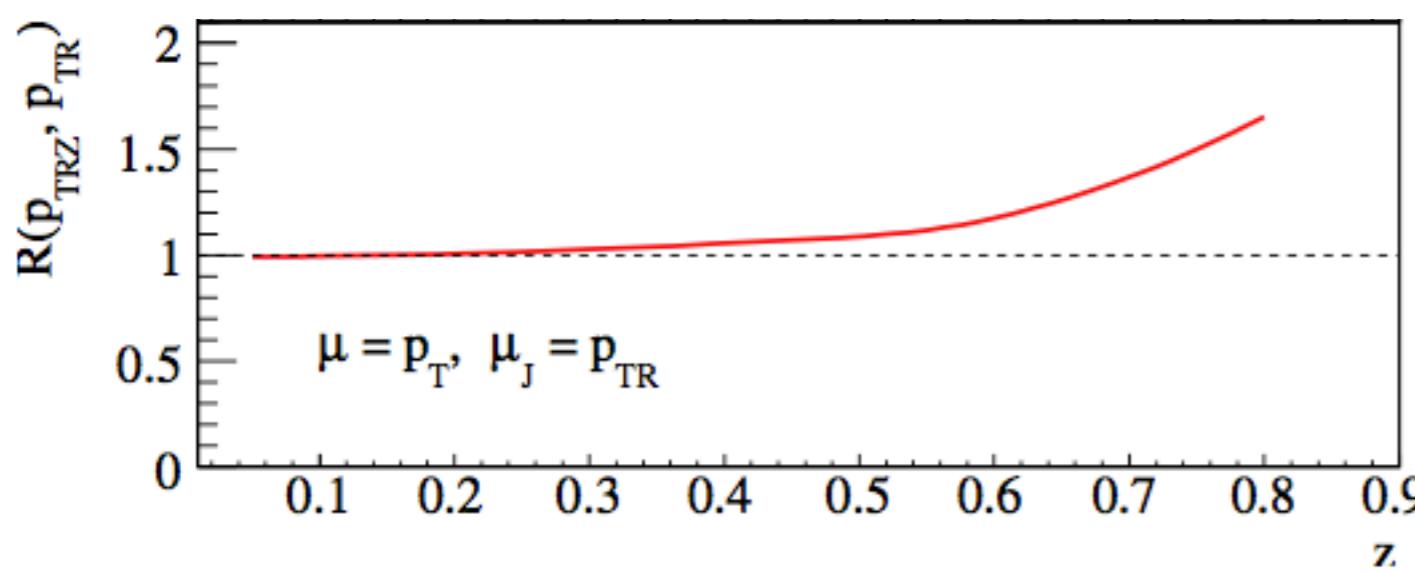
Hadronic threshold $z \rightarrow 1$

QCD scale uncertainty

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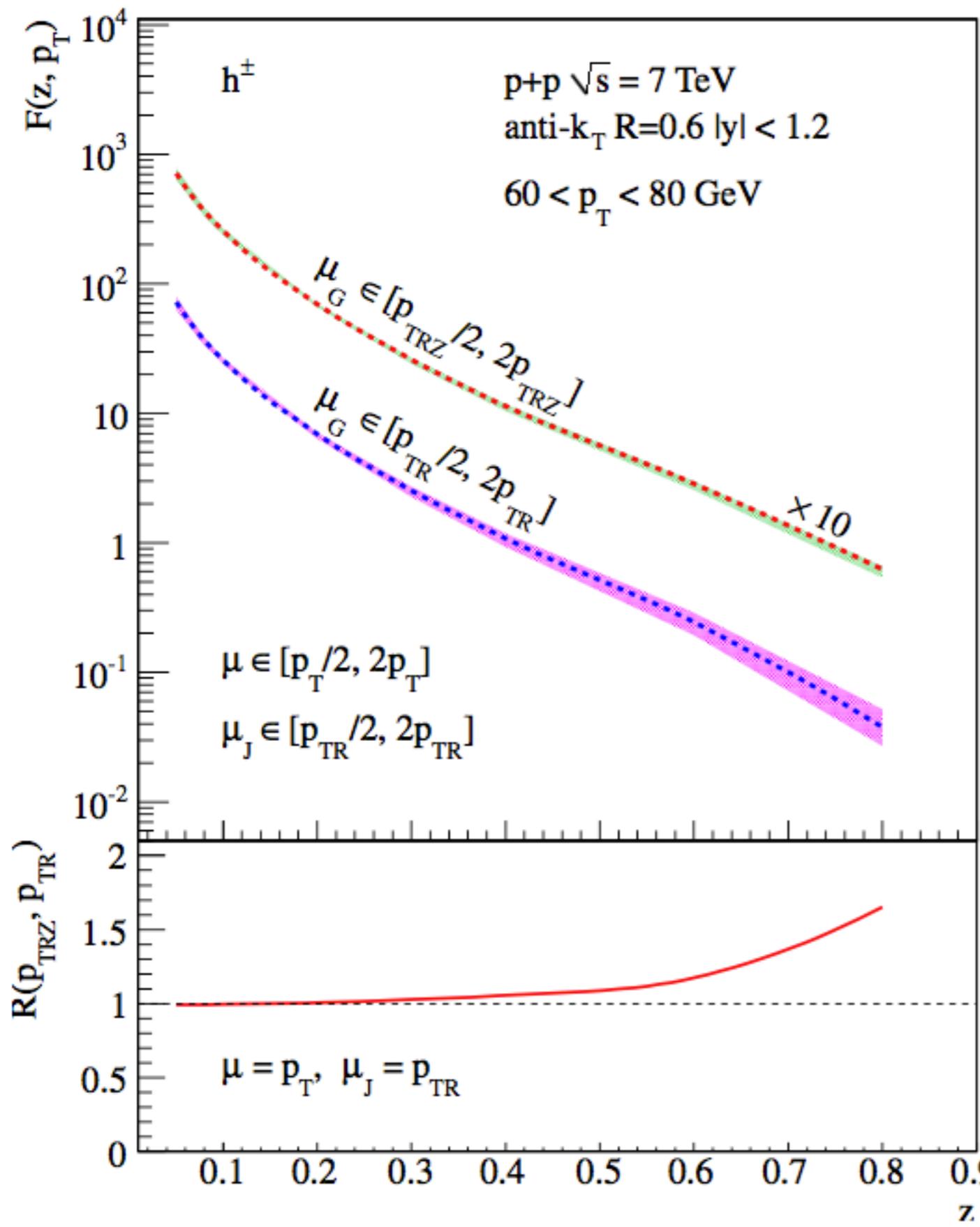
$$\mu_J \in [p_{TR}/2, 2p_{TR}],$$

$$\mu_g \in [p_{TRZ}/2, 2p_{TRZ}]$$



$$p_{TR} = 2p_T \tan(R/2)$$

$$p_{TRZ} = 2p_T \tan(R/2) (1 - z)$$



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Hadronic threshold $z \rightarrow 1$

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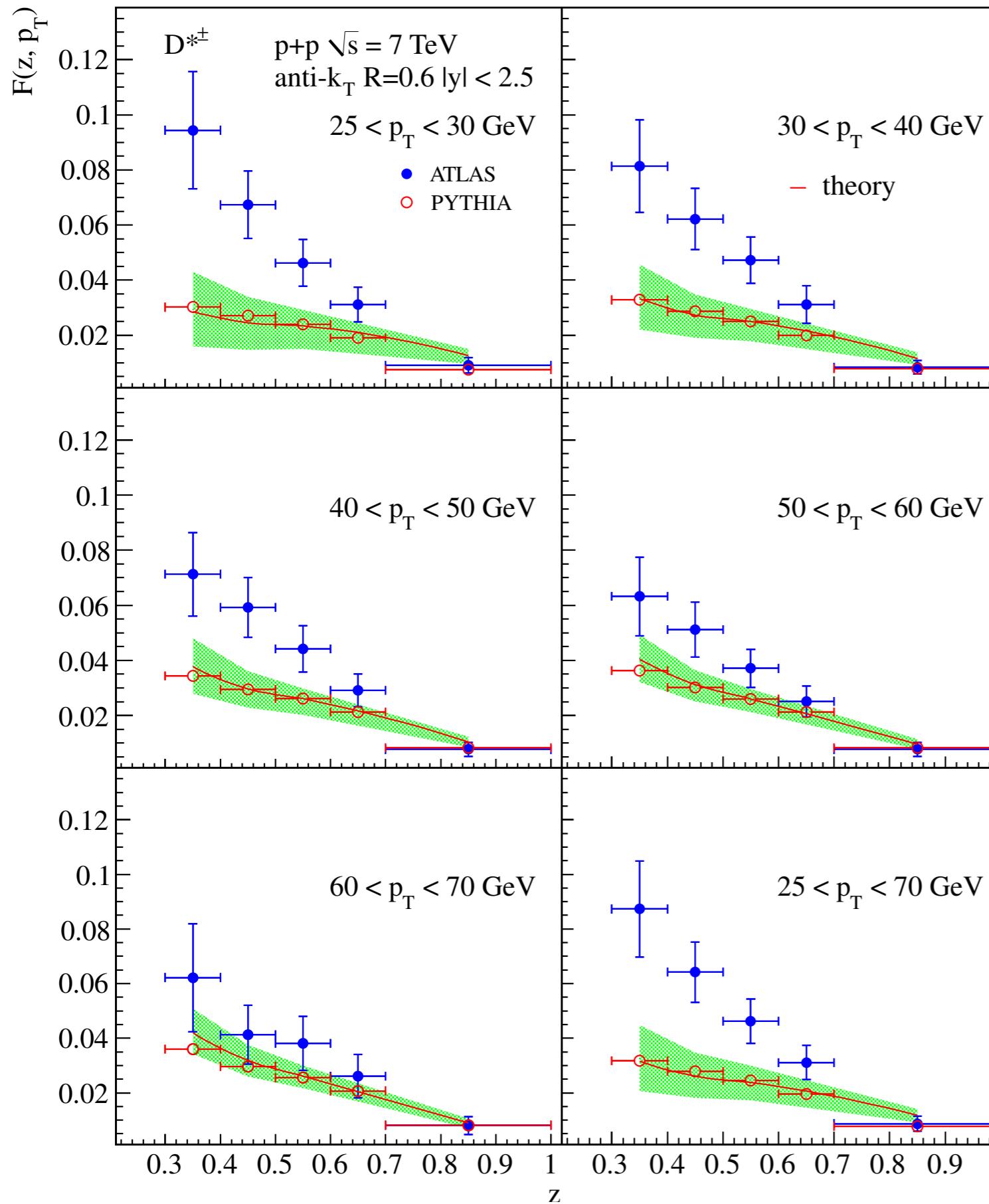
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$$p_{TRZ} = 2p_T \tan(R/2)(1 - z)$$

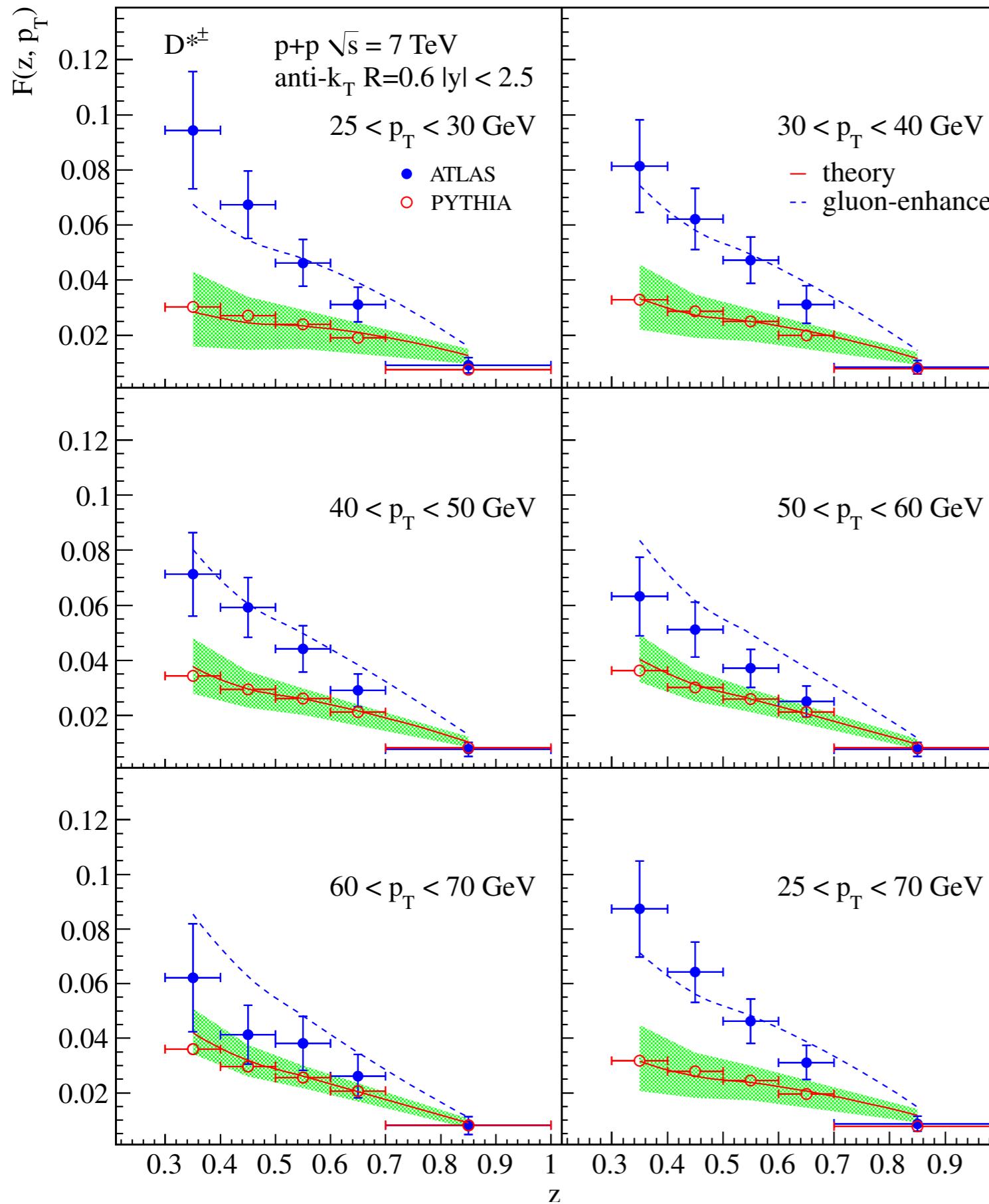


D-meson
jet fragmentation function

Comparison to ATLAS data
and PYTHIA simulations
at $\sqrt{s} = 7 \text{ TeV}$

Using FFs from
Kneesch, Kniehl, Kramer, Schienbein - '08

ZMVNFS, $e^+e^- \rightarrow DX$
 $\mu, \mu_J, \mu_G \gg m_Q$



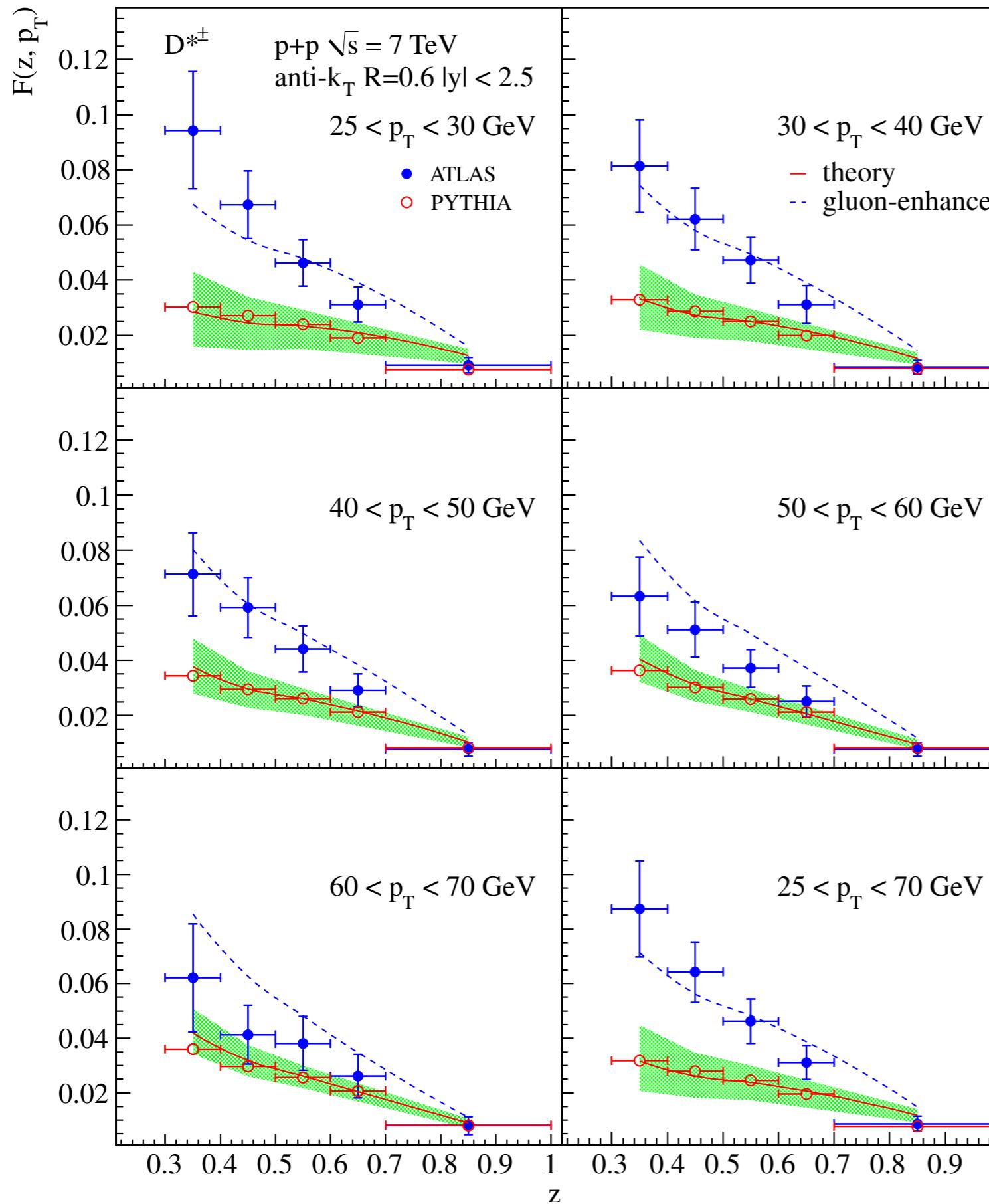
D-meson
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$$\text{--- --- } D_g^D(z, \mu) \rightarrow 2D_g^D(z, \mu)$$

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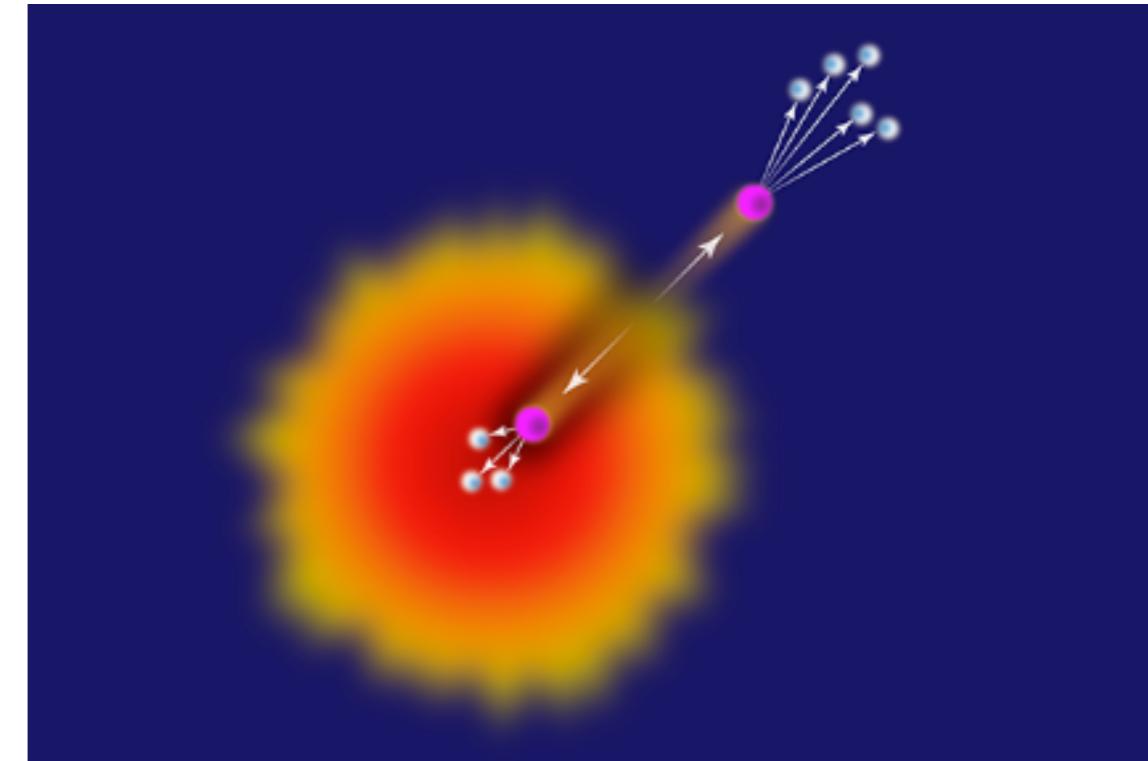
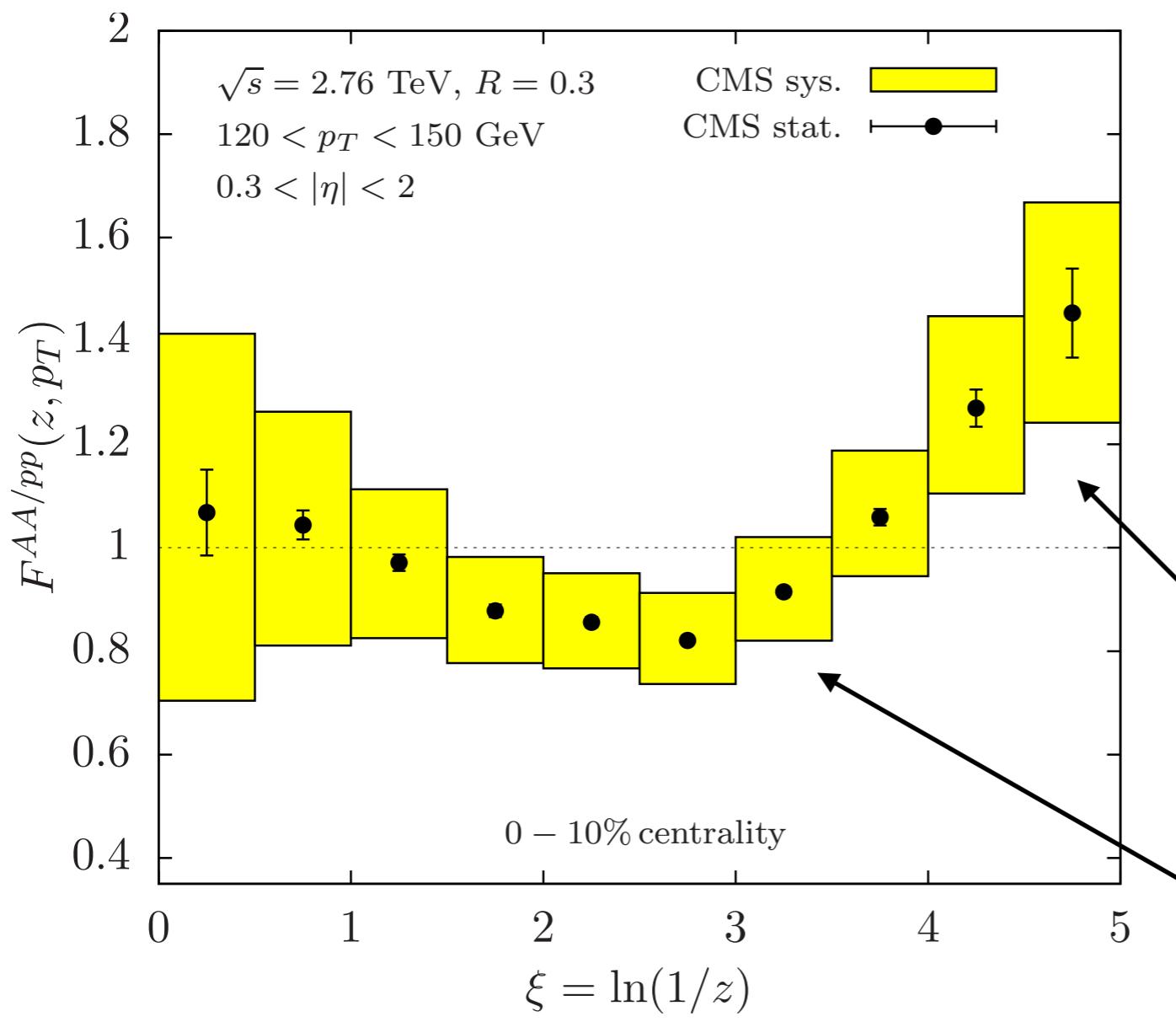
Comparison to ATLAS data
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New fit of D-FFs:
Anderle, Kang, FR, Stratmann, Vitev
- work in progress

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$$\frac{\frac{d\sigma_{AA}}{dydp_T}}{\langle N_{\text{bin}} \rangle \frac{d\sigma_{pp}}{dydp_T}}$$



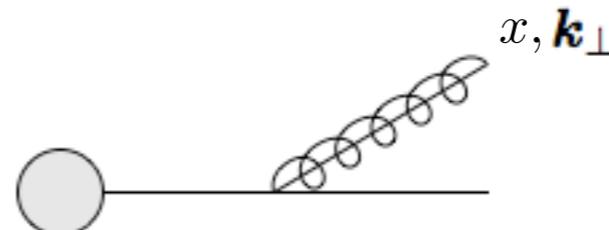
enhancement of soft particles,
small- z region

attenuation of particles in the
large- z region

SCE_G splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

I. Final state

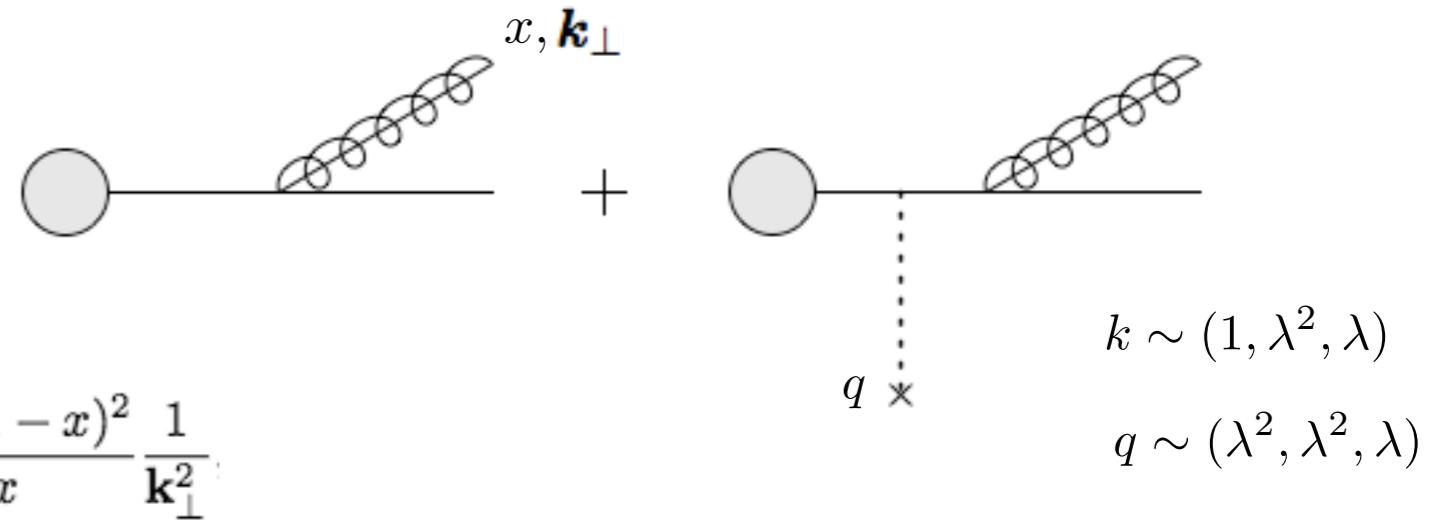


vacuum:
$$\left(\frac{dN^{\text{vac}}}{dx d^2\mathbf{k}_\perp} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1-x)^2}{x} \frac{1}{\mathbf{k}_\perp^2}$$

SCE_{T_G} splitting kernels

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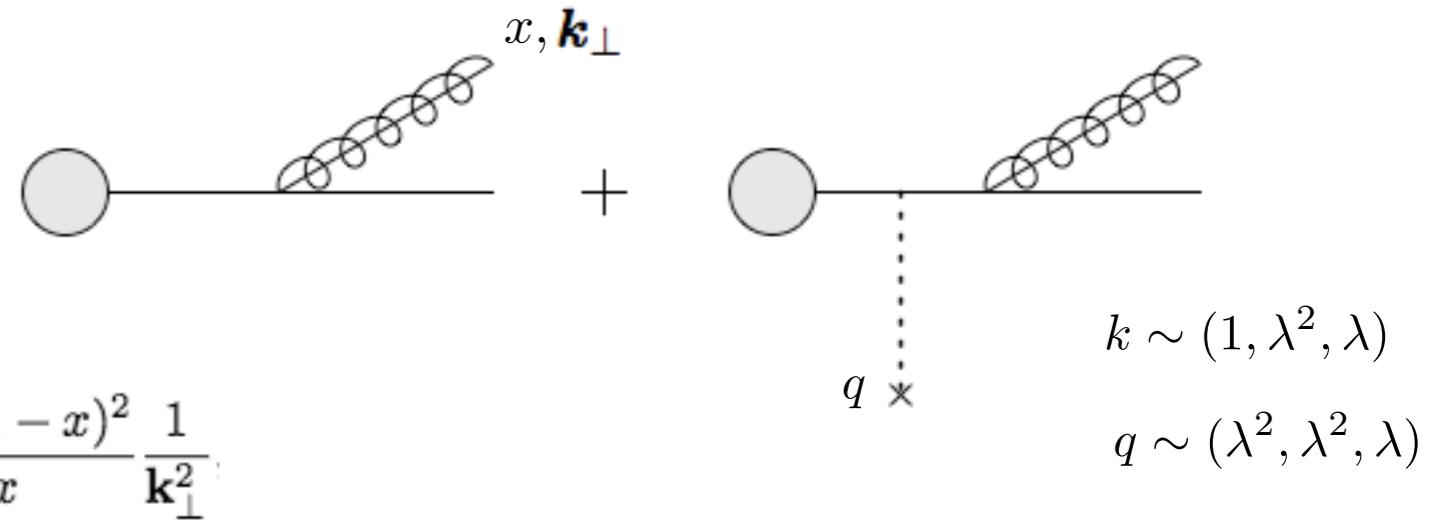
medium: Ovanesyan, Vitev '12 Idilbi, Majumder '08, D'Eramo, Liu, Rajagopal '10

$$\mathcal{L}_{\text{SCET}_G} = \mathcal{L}_{\text{SCET}} + \mathcal{L}_G(\xi_n, A_n, A_G)$$

SCE_G splitting kernels

Basic ingredients for the calculation
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vacuum: $\left(\frac{dN^{\text{vac}}}{dx d^2\mathbf{k}_\perp} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1-x)^2}{x} \frac{1}{\mathbf{k}_\perp^2}$

medium: Ovanesyan, Vitev '12 Idilbi, Majumder '08, D'Eramo, Liu, Rajagopal '10

$$\left(\frac{dN^{\text{med}}}{dx d^2\mathbf{k}_\perp} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi^2} C_F \frac{1 + (1-x)^2}{x} \int \frac{d\Delta z}{\lambda_g(z)} \int d^2\mathbf{q}_\perp \frac{1}{\sigma_{el}} \frac{d\sigma_{el}^{\text{medium}}}{d^2\mathbf{q}_\perp} \left[\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2} \cdot \left(\frac{\mathbf{B}_\perp}{\mathbf{B}_\perp^2} - \frac{\mathbf{C}_\perp}{\mathbf{C}_\perp^2} \right) (1 - \cos[(\Omega_1 - \Omega_2)\Delta z]) + \dots \right]$$

model dependence of the medium

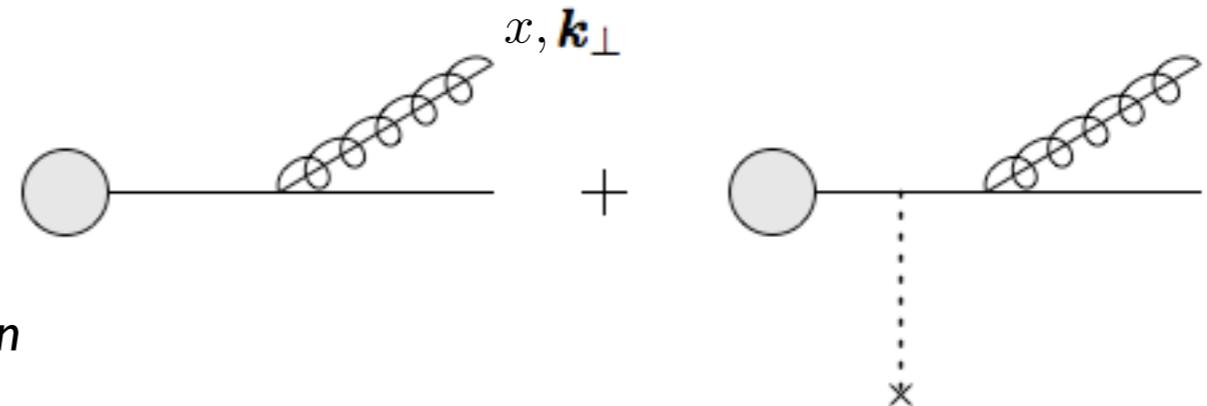
Soft gluon limit is consistent with traditional energy loss approach

Gyulassy, Levai, Vitev '00

SCE_G splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

I. Final state *Ovanesyan, Vitev '12*



2. Final state - massive *FR, Vitev - in preparation*

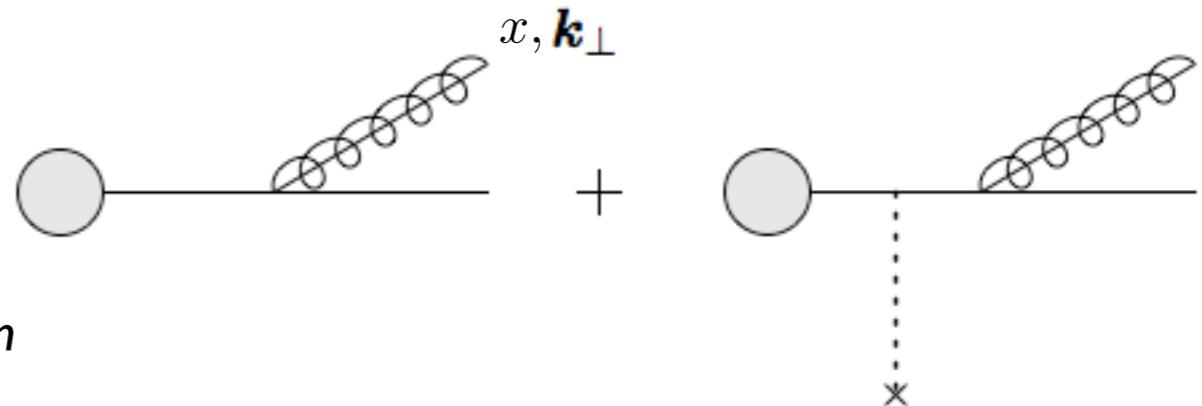
e.g. vacuum

$$\left(\frac{dN}{dxd^2k_\perp} \right)_{Q \rightarrow Qg} = C_F \frac{\alpha_s}{\pi^2} \frac{1}{k_\perp^2 + x^2m^2} \left[\frac{1-x+x^2/2}{x} - \frac{x(1-x)m^2}{k_\perp^2 + x^2m^2} \right]$$

SCE T_G splitting kernels

Basic ingredients for the calculation
of the modification in AA collisions:

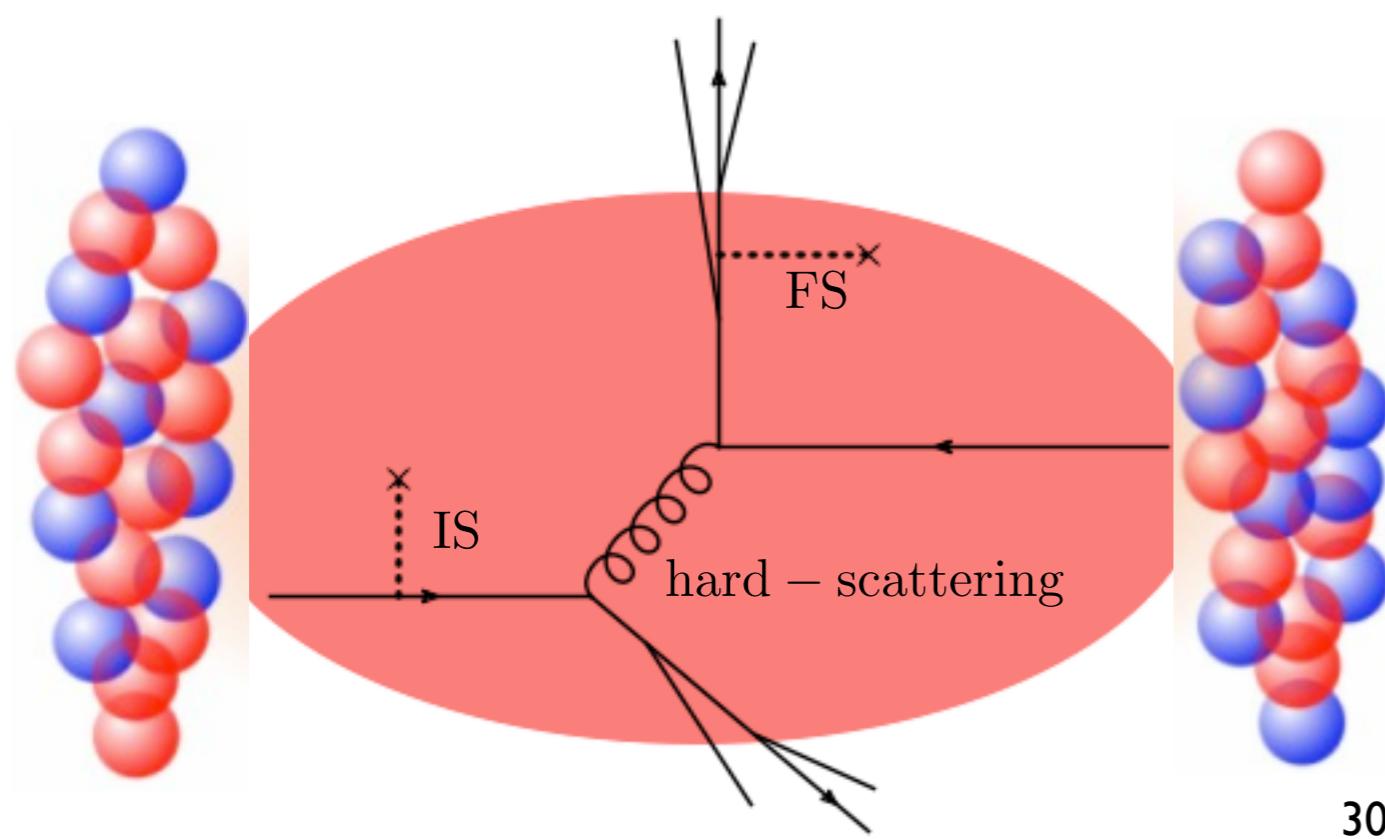
1. Final state *Ovanesyan, Vitev '12*



2. Final state - massive *FR, Vitev - in preparation*

3. Initial state *Ovanesyan, FR, Vitev - '15*

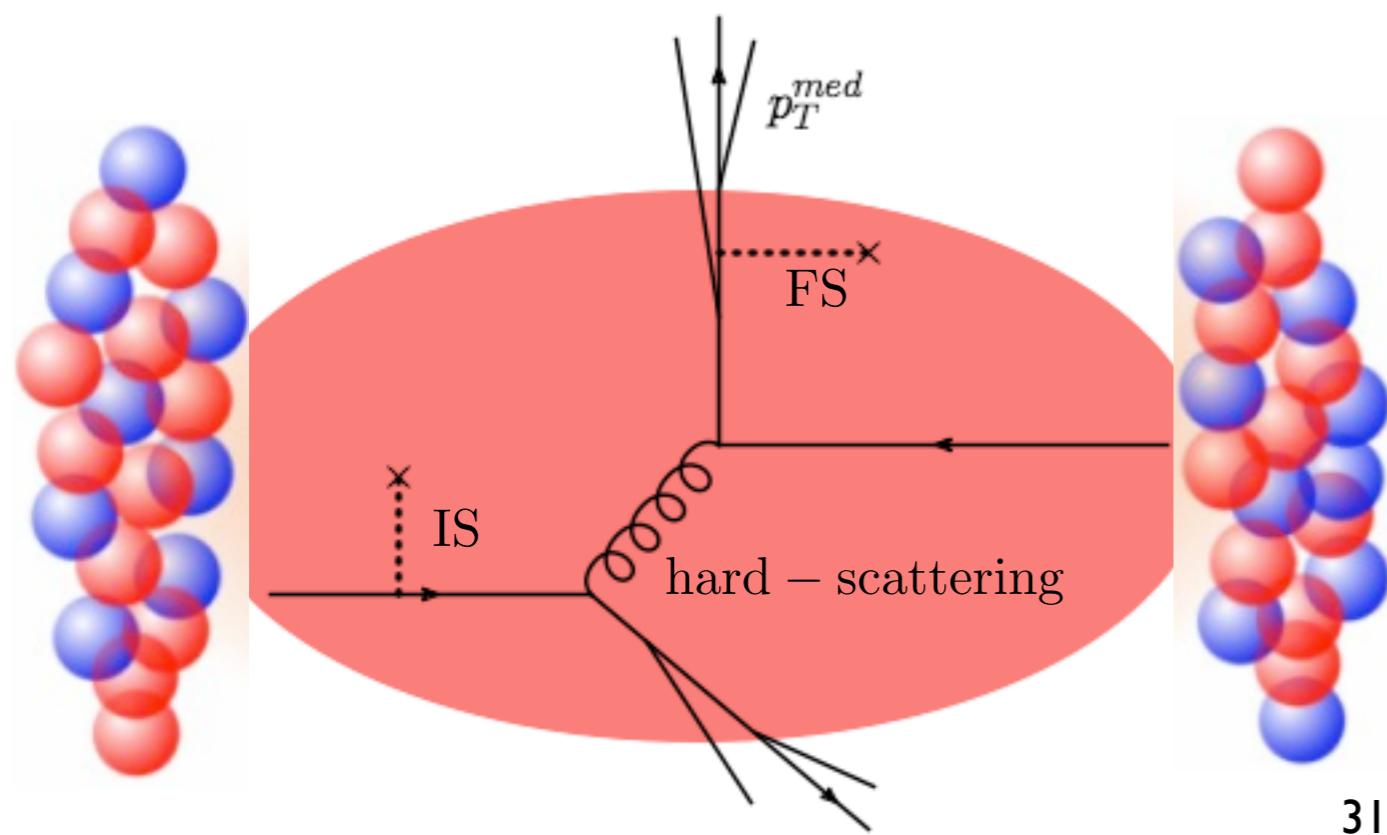
I. Initial state - Cold Nuclear Matter (CNM)



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$$\sum_{i=q,g} \frac{d\sigma_{AA}^i}{d\eta dp_T} \frac{\mathcal{G}_i^{j,AA}}{J_i^{AA}}$$

2. Final state - jet energy loss



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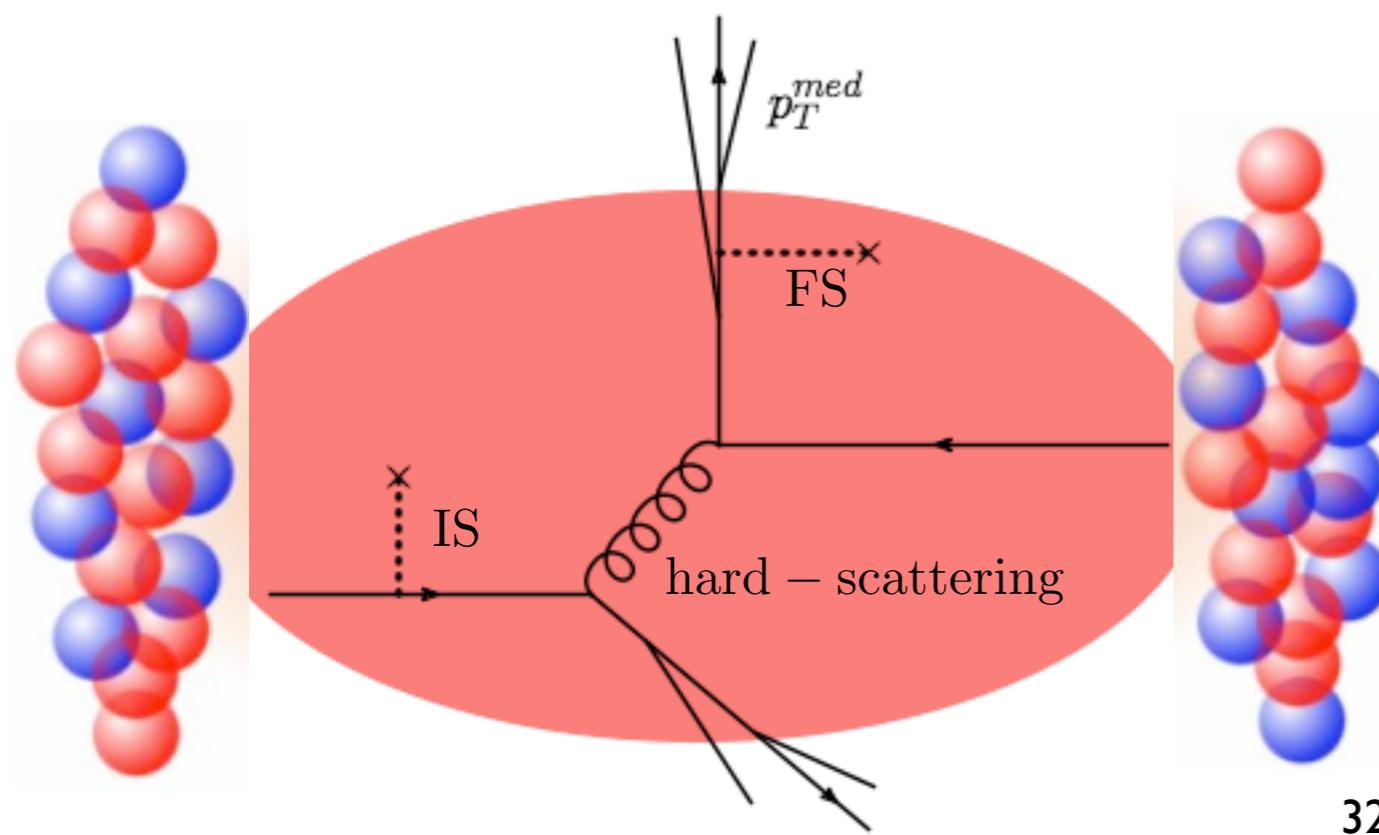
Chien, Vitev '15
He, Vitev, Zhang '12
Dasgupta, Dreyer, Salam, Soyez '15

I. Initial state - Cold Nuclear Matter (CNM)

$$\sum_{i=q,g} \frac{d\sigma_{AA}^i}{d\eta dp_T} \frac{\mathcal{G}_i^{j,AA}}{J_i^{AA}}$$

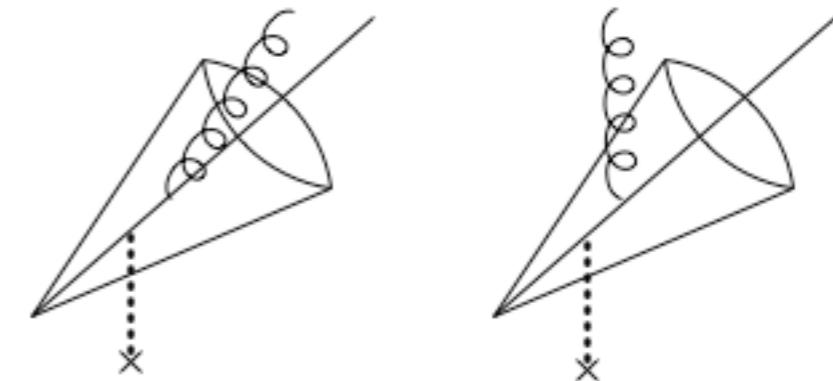
$$p_T^{med} = p_T^{vac} \times (1 - \epsilon) \quad \longrightarrow \quad \frac{1}{\sigma_0} \frac{d\sigma_{AA}^i}{dy dp_T} \Big|_{p_T} = \frac{1}{\sigma_0} \frac{d\sigma_{pp}^i}{dy dp_T} \Big|_{\frac{p_T}{1-\epsilon_i}} \frac{1}{1 - \epsilon_i}$$

\longrightarrow Inclusive jet R_{AA}



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2. Final state - jet energy loss



Chien, Vitev '15
He, Vitev, Zhang '12
Dasgupta, Dreyer, Salam, Soyez '15

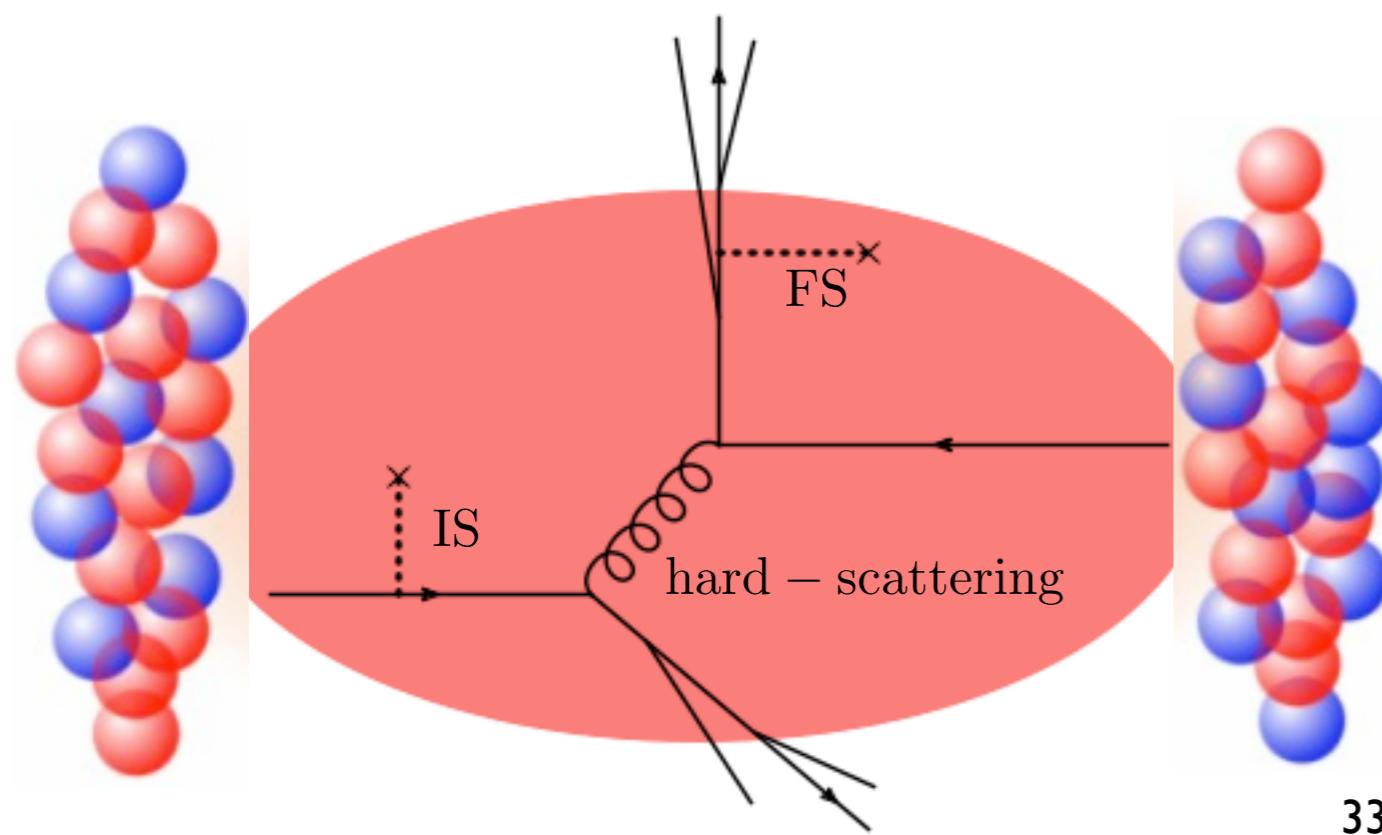
I. Initial state - Cold Nuclear Matter (CNM)

3. Final state - jet substructure

2. Final state - jet energy loss

$$\mathcal{G}_{i,\text{bare}}^j(\omega, R, z, \mu) = \int dk_{\perp}^2 \frac{(e^{\gamma_E} \mu^2)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{dN^{\text{vac}}}{dx dk_{\perp}^2} \right)_{i \rightarrow jk} \delta_{\text{alg}} \quad (x \rightarrow 1-z)$$

$$\left(\frac{dN^{\text{vac}}}{dx dk_{\perp}^2} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi} C_F \frac{1 + (1-x)^2}{x} \frac{1}{(k_{\perp}^2)^{1+\epsilon}}$$



I. Initial state - Cold Nuclear Matter (CNM)

3. Final state - jet substructure

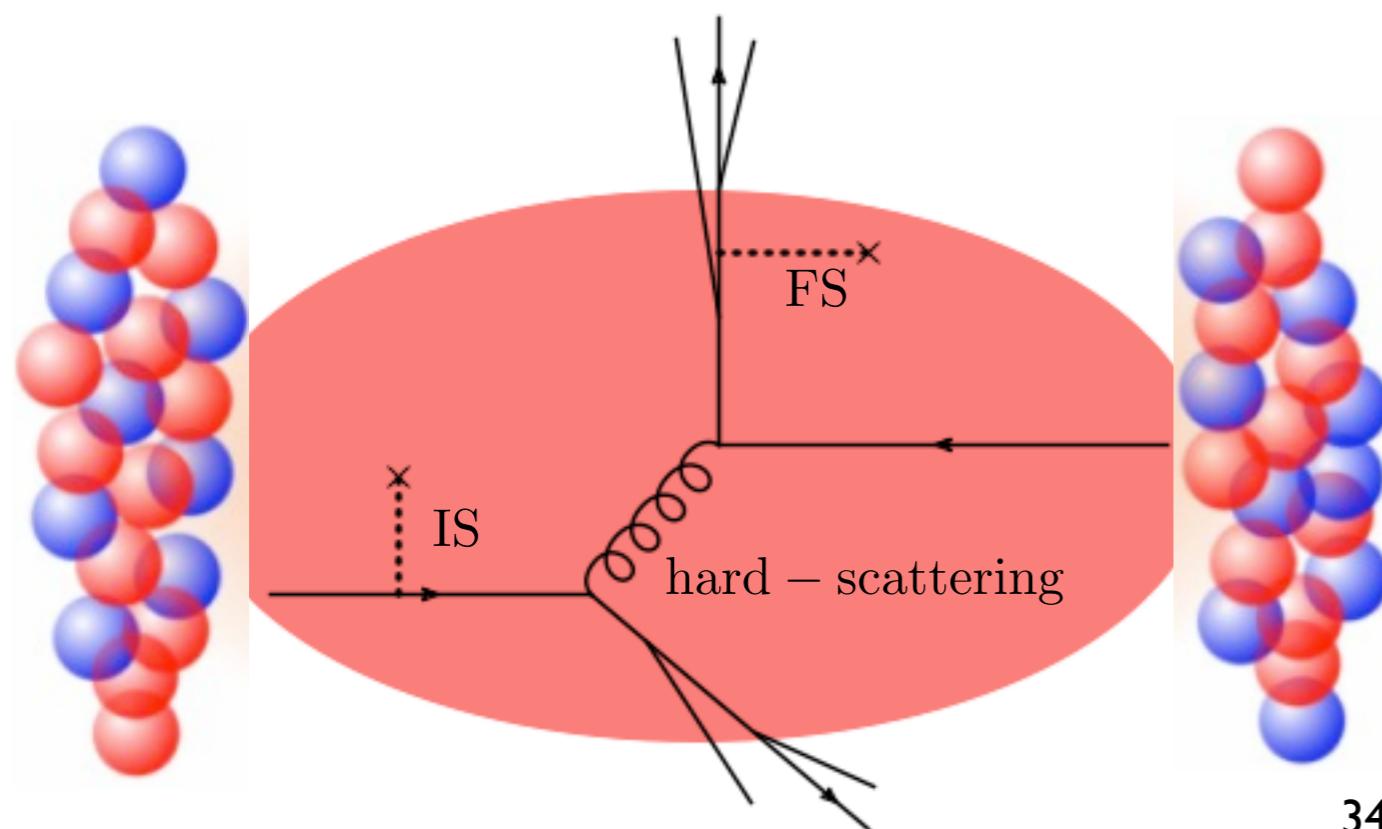
$$\mathcal{G}_{i,\text{bare}}^j(\omega, R, z, \mu) = \int dk_{\perp}^2 \frac{(e^{\gamma_E} \mu^2)^{\epsilon}}{\Gamma(1-\epsilon)} \left(\frac{dN^{\text{vac}}}{dx dk_{\perp}^2} \right)_{i \rightarrow jk} \delta_{\text{alg}} \quad (x \rightarrow 1-z)$$

↗

$$\left(\frac{dN^{\text{vac}}}{dx dk_{\perp}^2} \right)_{q \rightarrow qg} = \frac{\alpha_s}{2\pi} C_F \frac{1 + (1-x)^2}{x} \frac{1}{(k_{\perp}^2)^{1+\epsilon}}$$

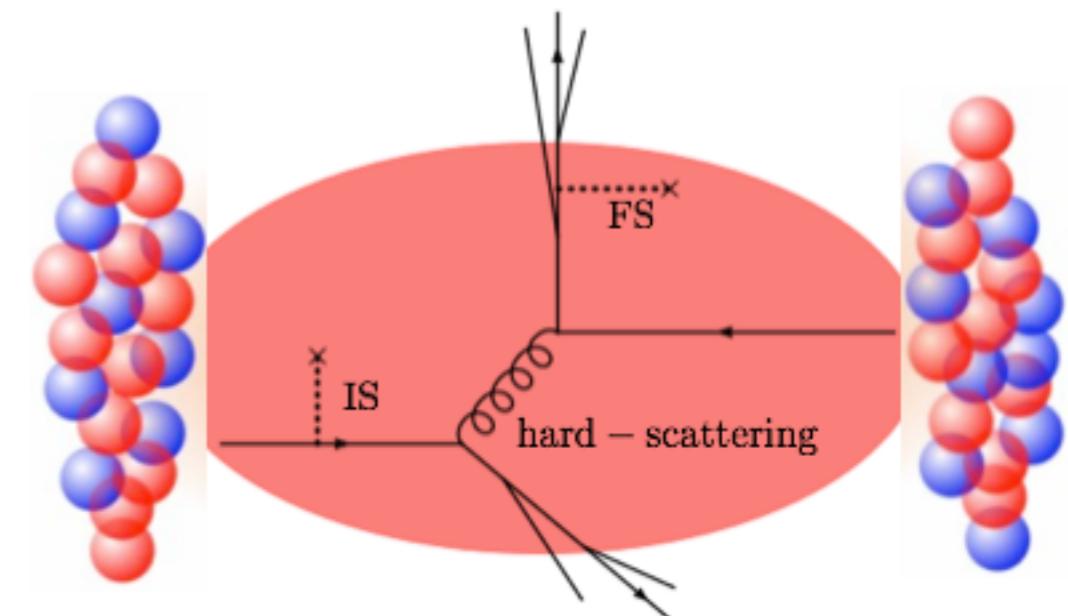
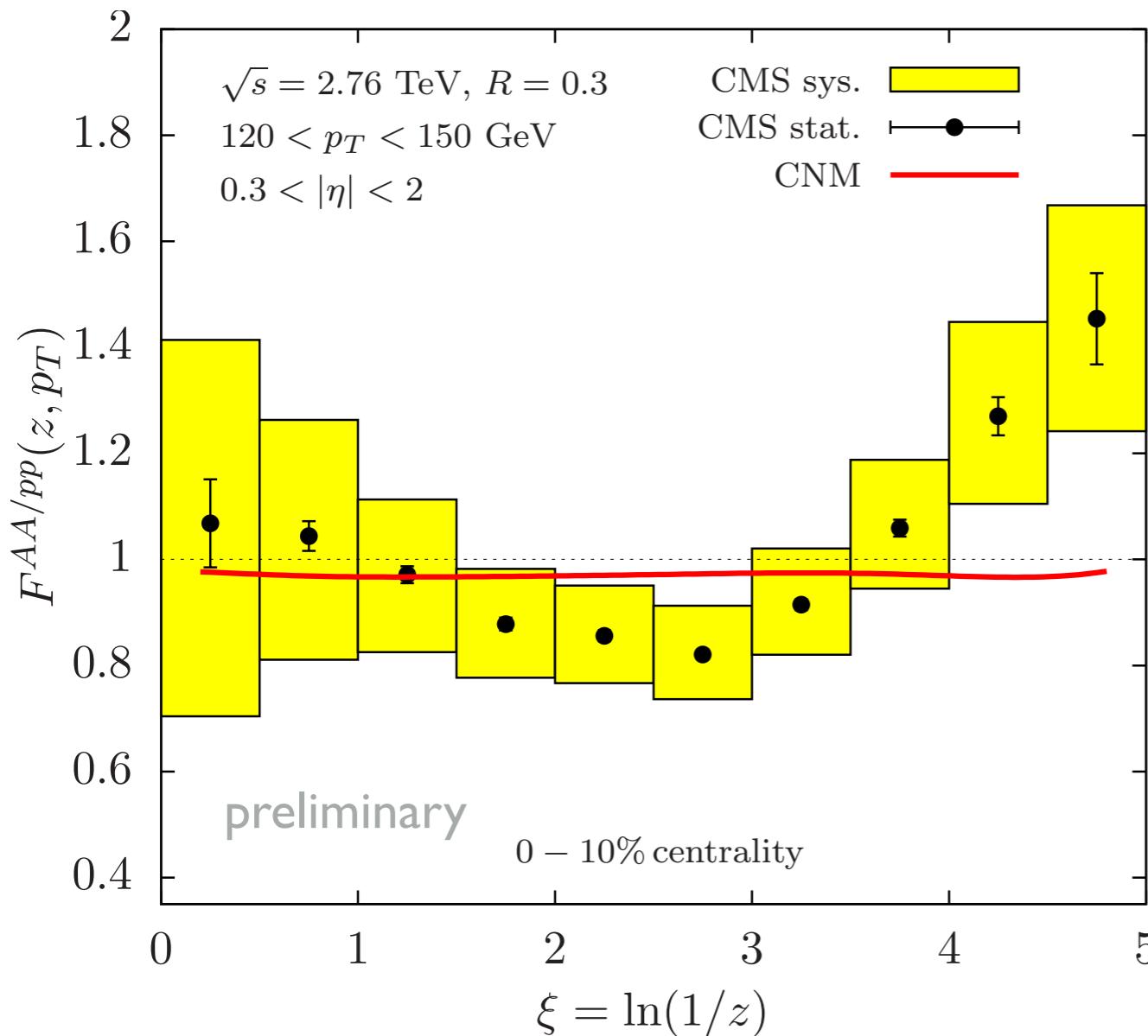
→

$$\mathcal{G}_{i,\text{bare}}^{j,\text{AA}}(\omega, R, z, \mu) = \int dk_{\perp}^2 \frac{(e^{\gamma_E} \mu^2)^{\epsilon}}{\Gamma(1-\epsilon)} \left[\left(\frac{dN^{\text{vac}}}{dx dk_{\perp}^2} \right)_{i \rightarrow jk} + \left(\frac{dN^{\text{med}}}{dx dk_{\perp}^2} \right)_{i \rightarrow jk} \right] \delta_{\text{alg}}$$

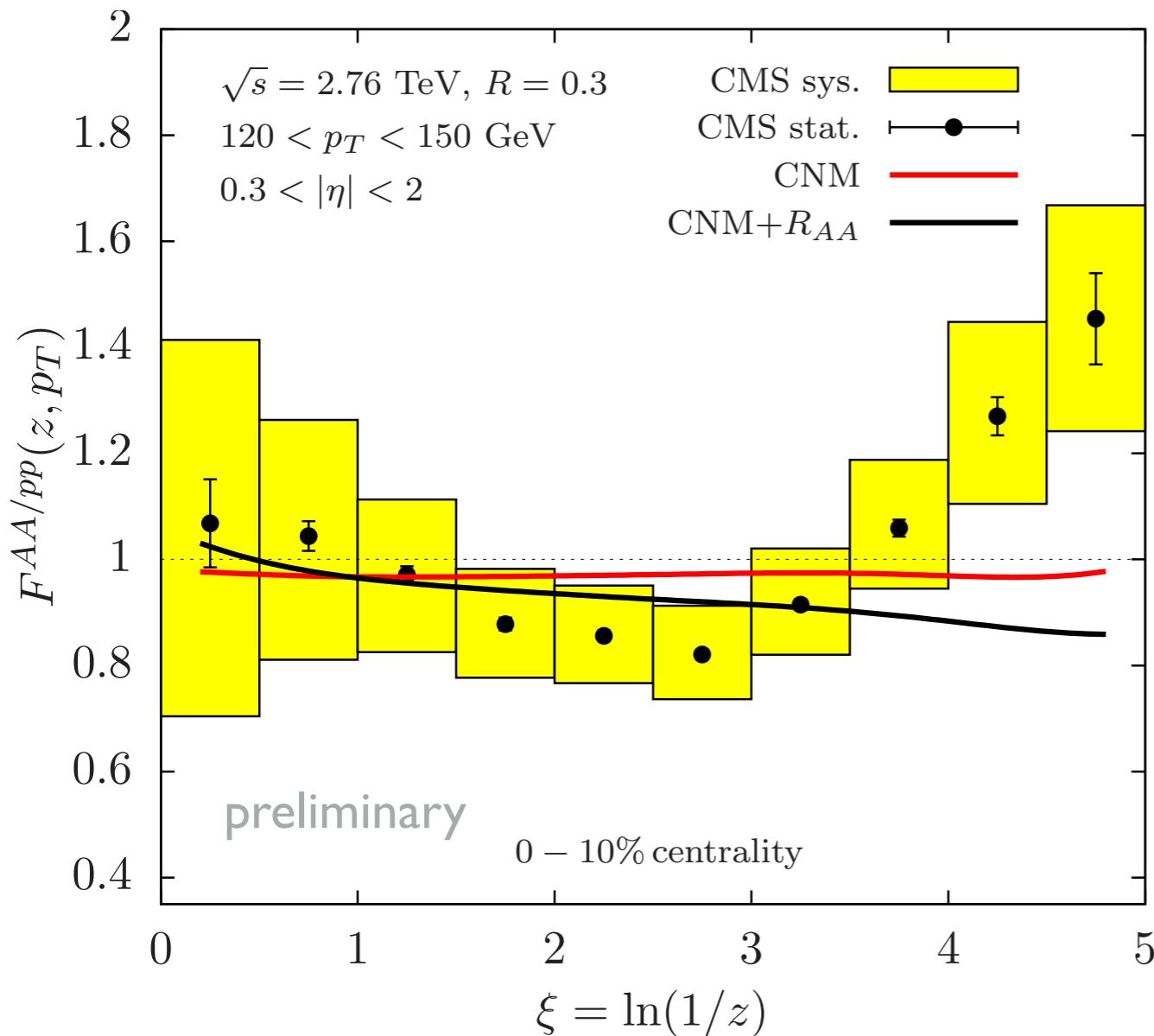


34

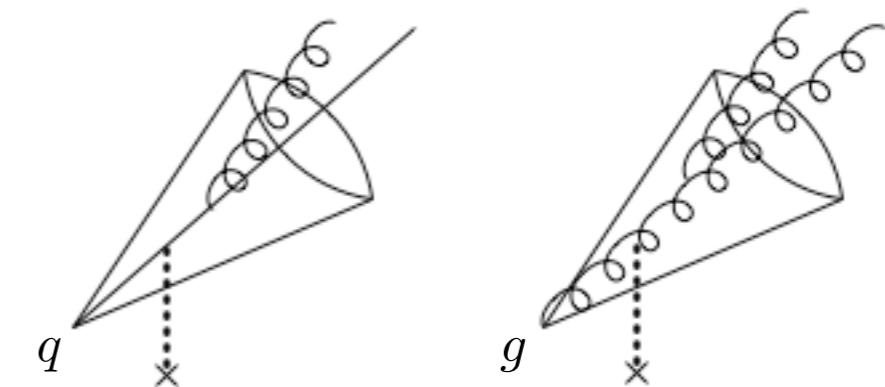
Similarly: $J_i^{\text{AA}}(\mu) = J_i^{\text{vac}}(\mu) + J_i^{\text{med}}(\mu)$



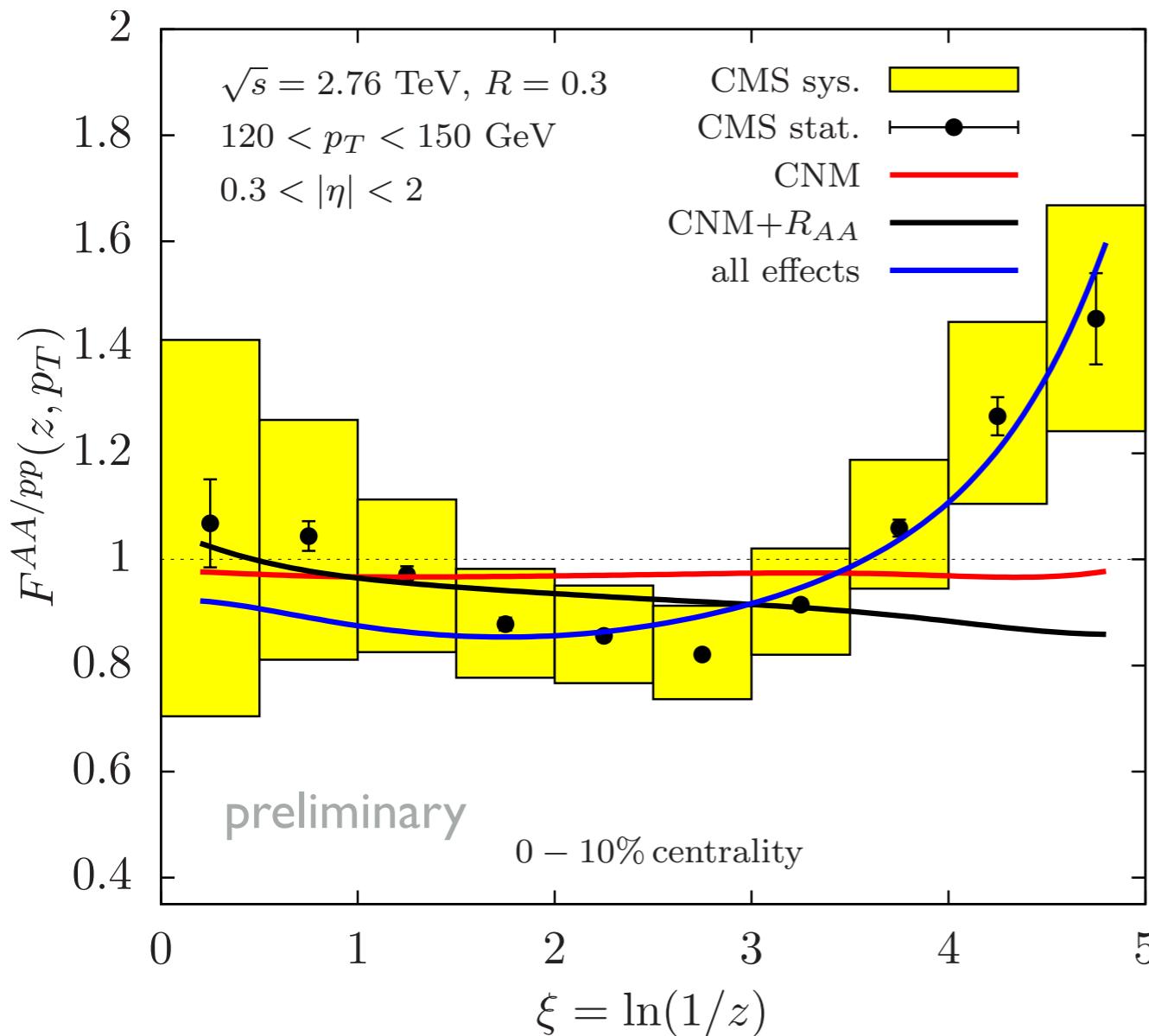
Using Kretzer FFs
Kretzer - '00



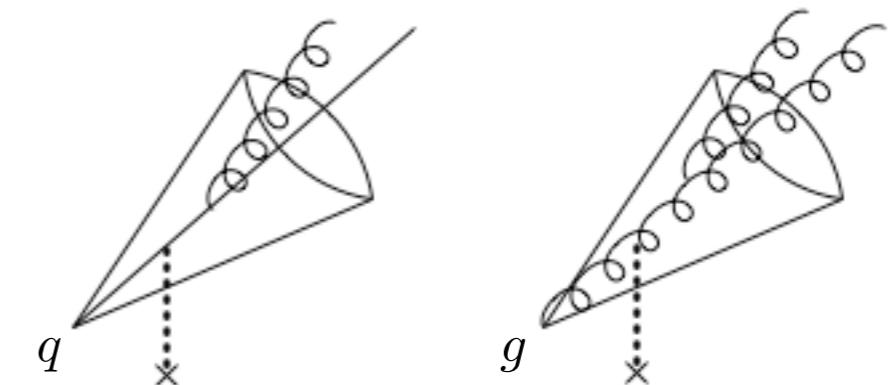
$$\sum_{i=q,g} \frac{d\sigma_{AA}^i}{d\eta dp_T}$$



Using Kretzer FFs
Kretzer - '00

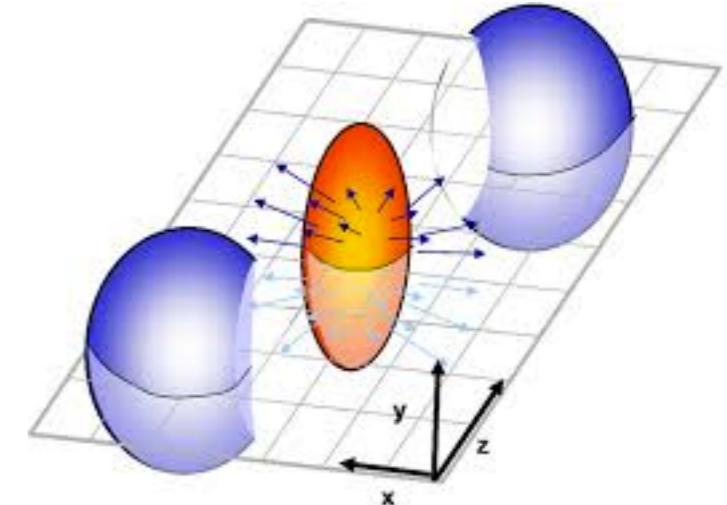
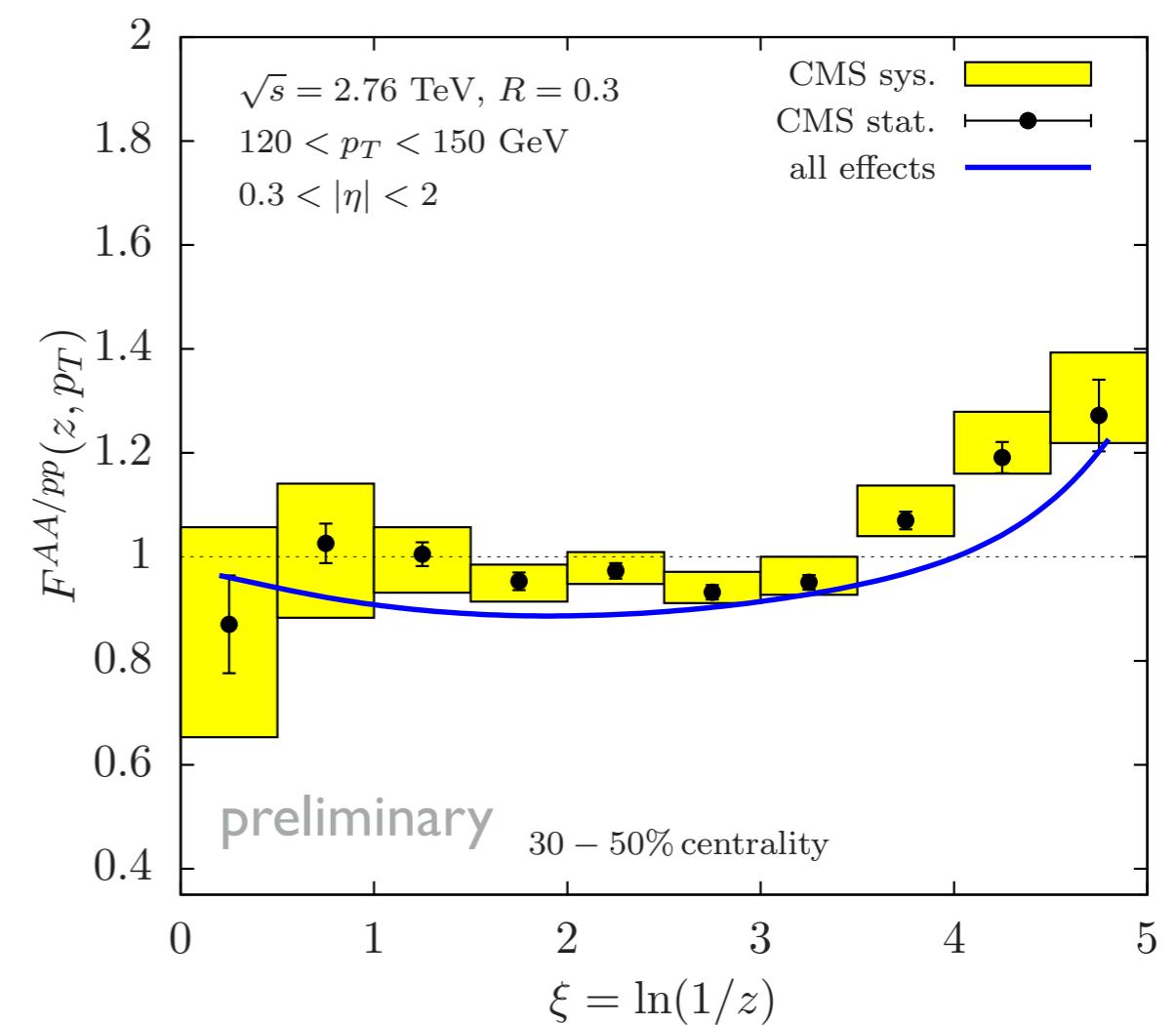
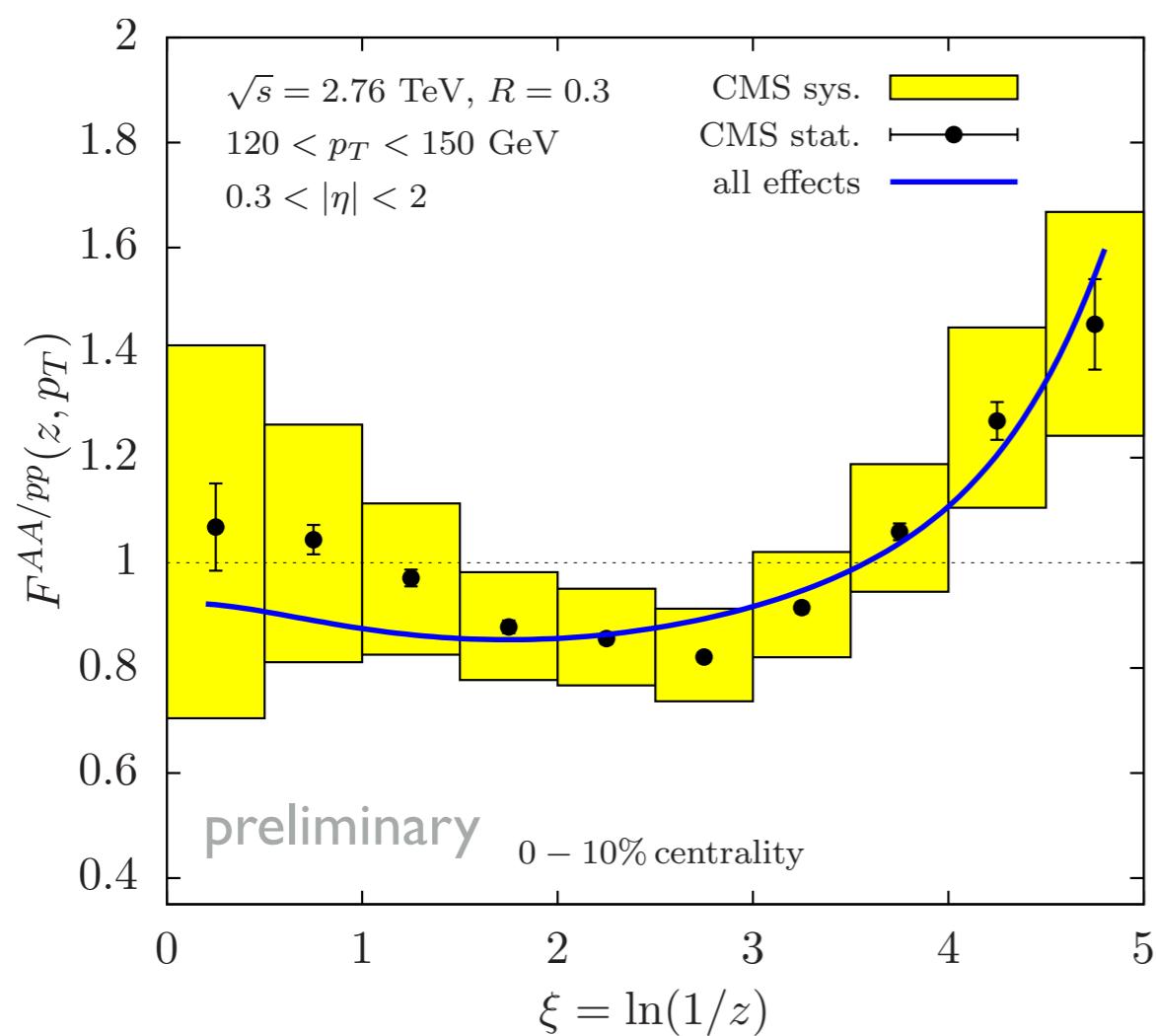


$$\sum_{i=q,g} \frac{d\sigma_{AA}^i}{d\eta dp_T} \frac{\mathcal{G}_i^{j,AA}}{J_i^{AA}}$$



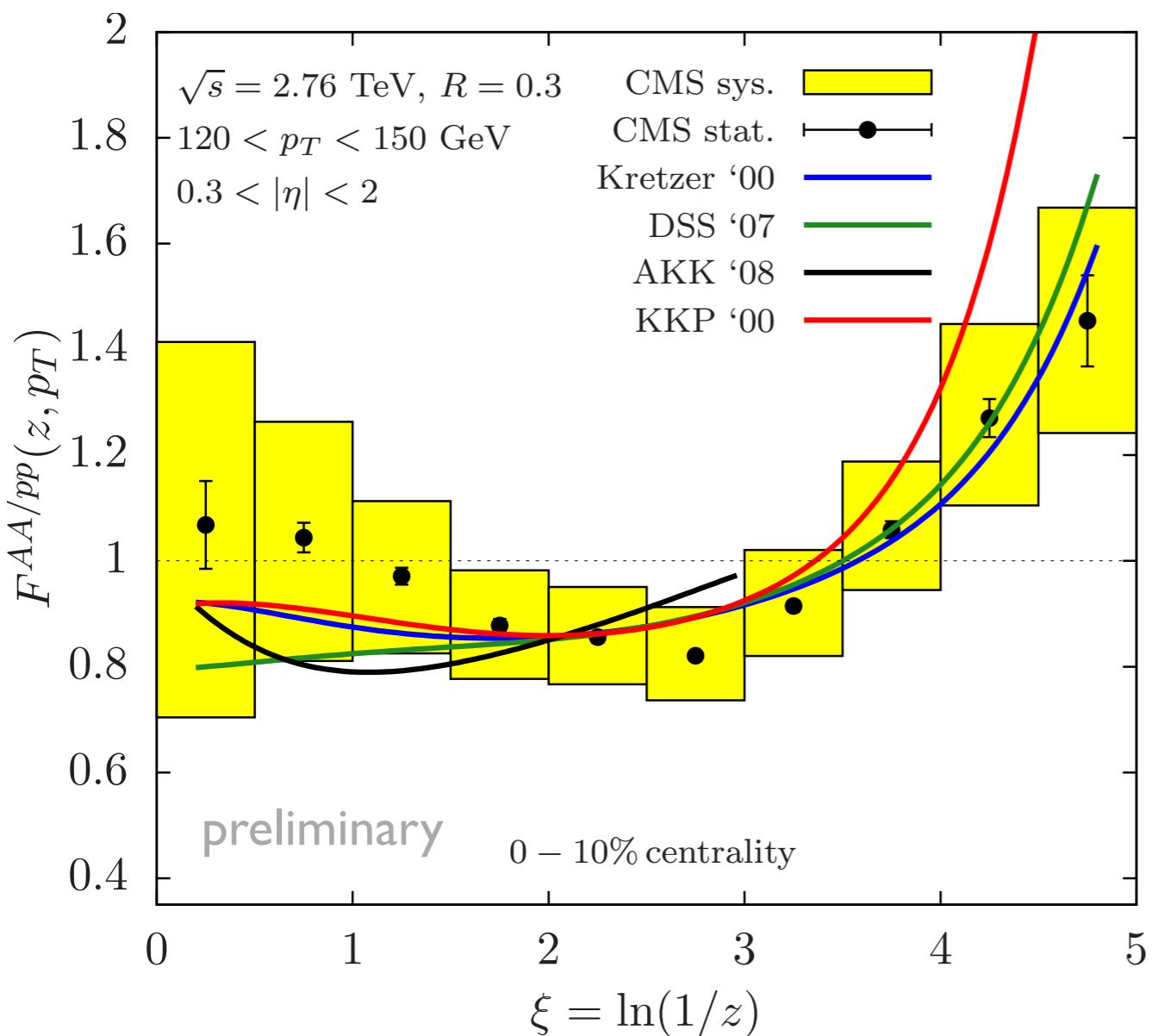
Using Kretzer FFs
Kretzer - '00

Centrality dependence



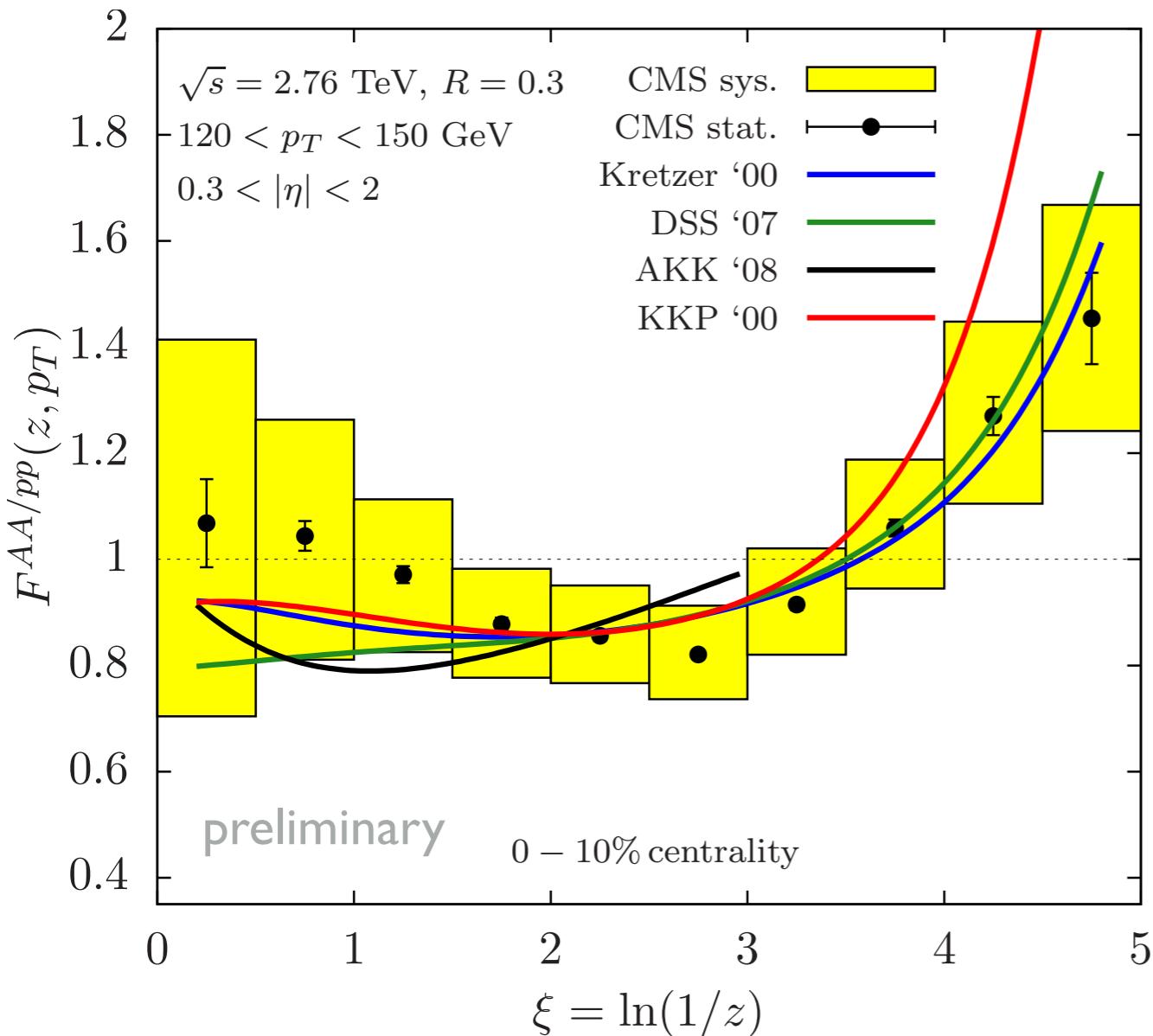
Uncertainties

- fragmentation functions

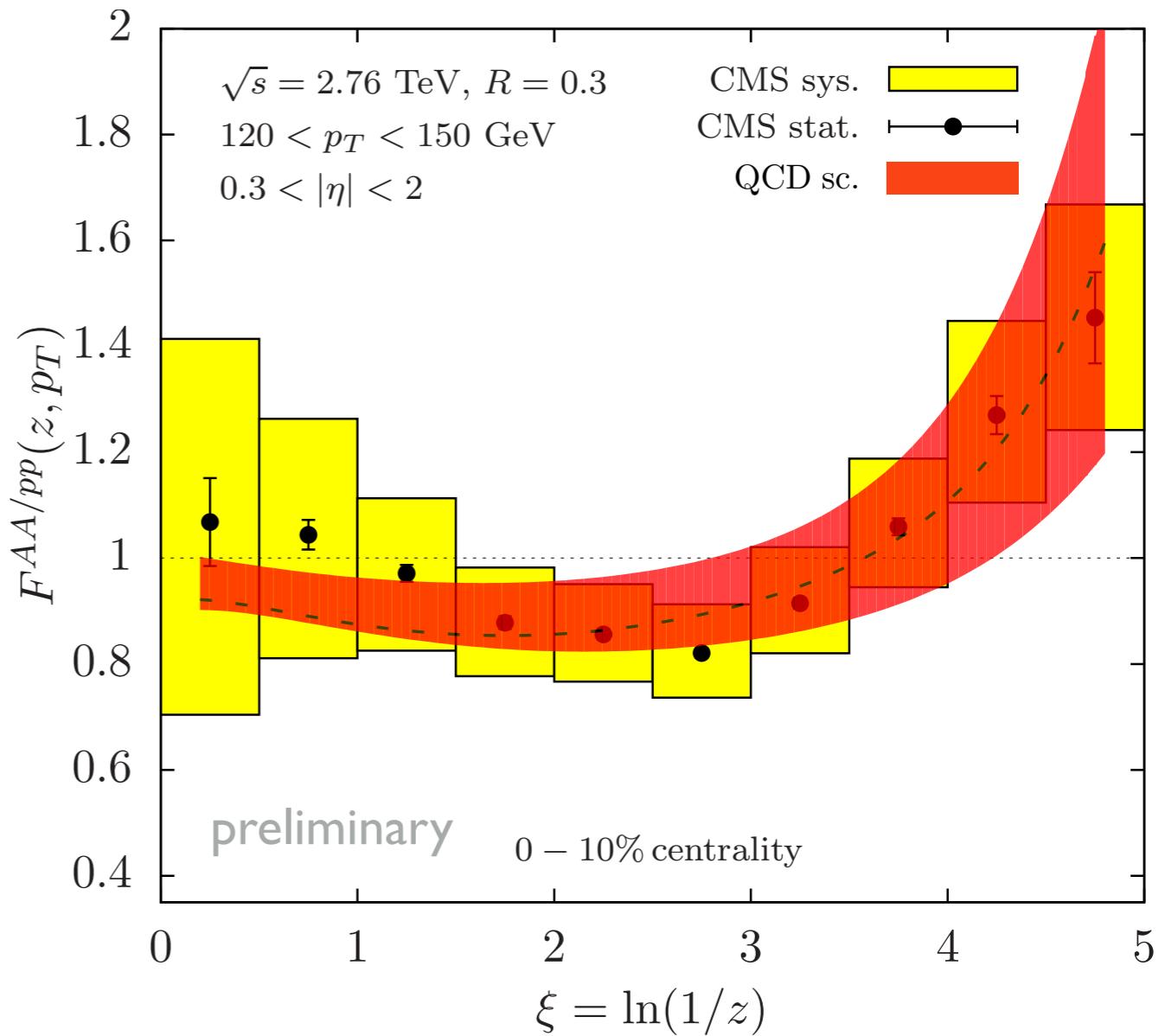


Uncertainties

- fragmentation functions



- QCD scale uncertainty μ, μ_G, μ_J

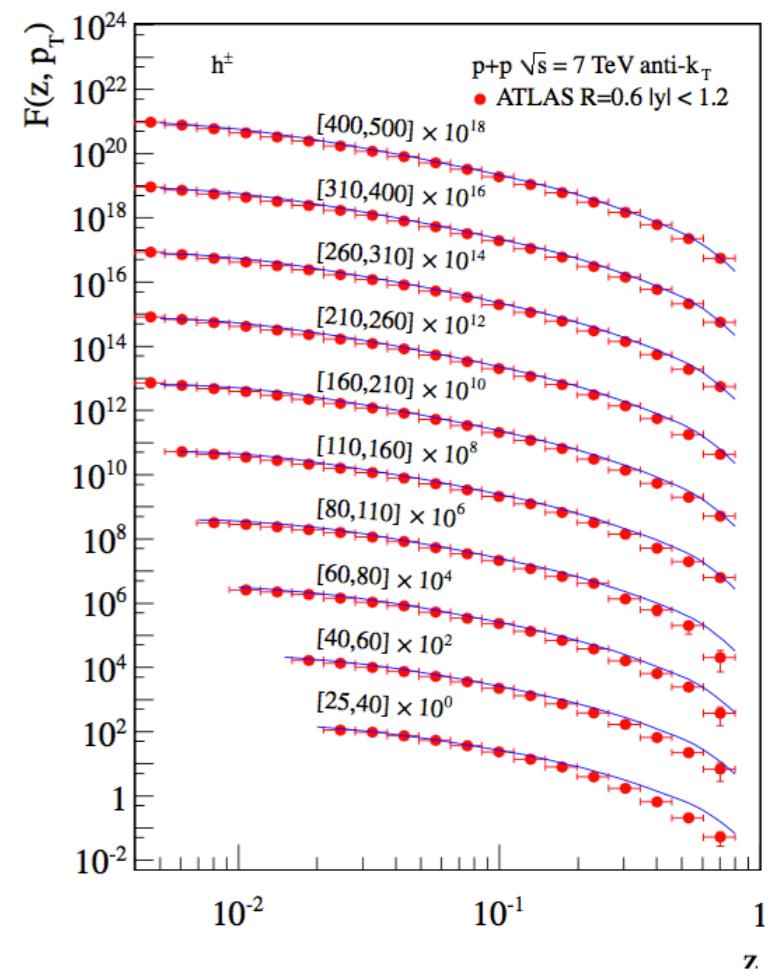


Outline

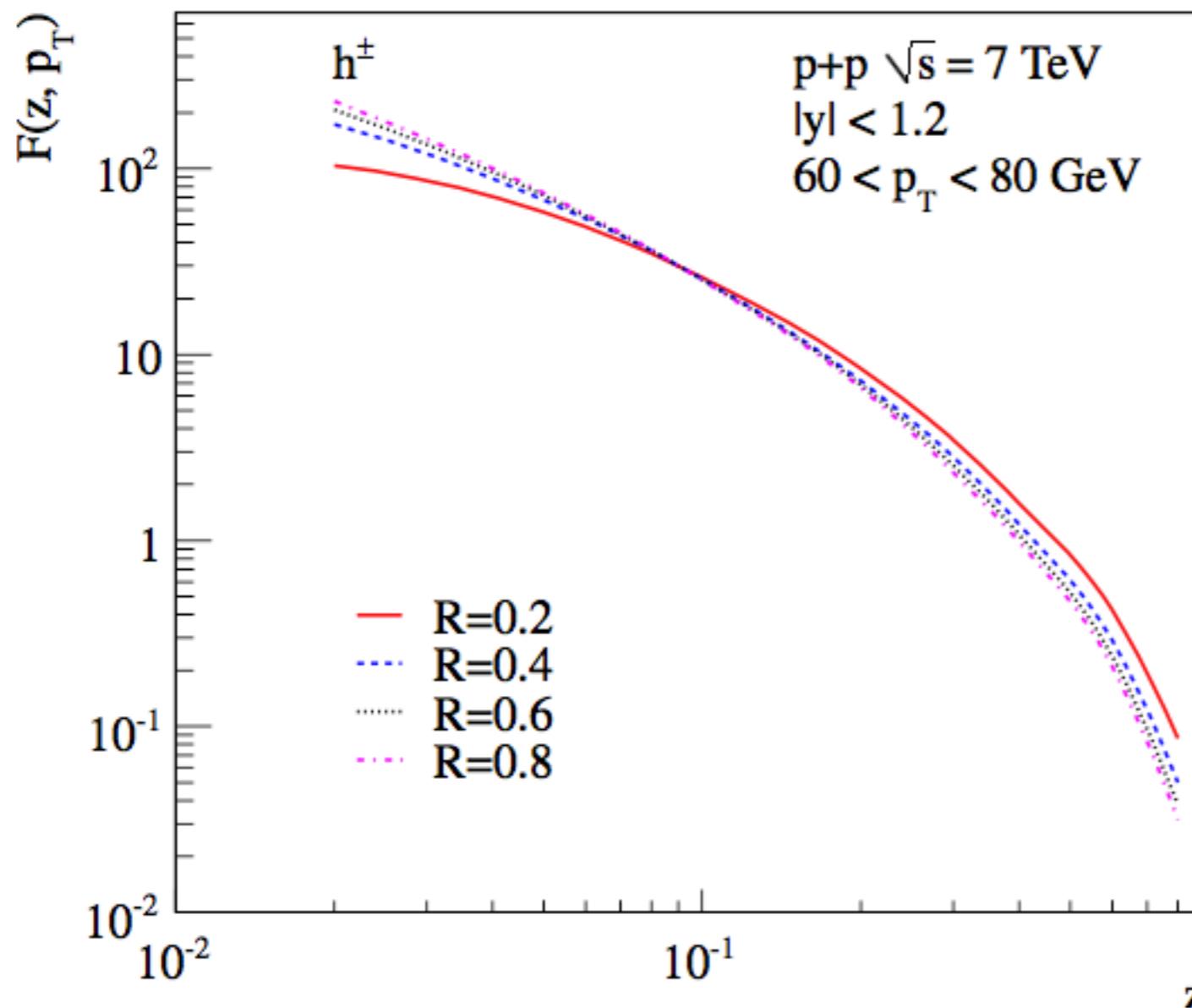
- Jet Fragmentation functions in pp collisions
Chien, Kang, FR, Vitev, Xing - '15
- Modification in AA
Chien, Kang, FR, Vitev, Xing - in preparation
- Conclusions

Conclusions

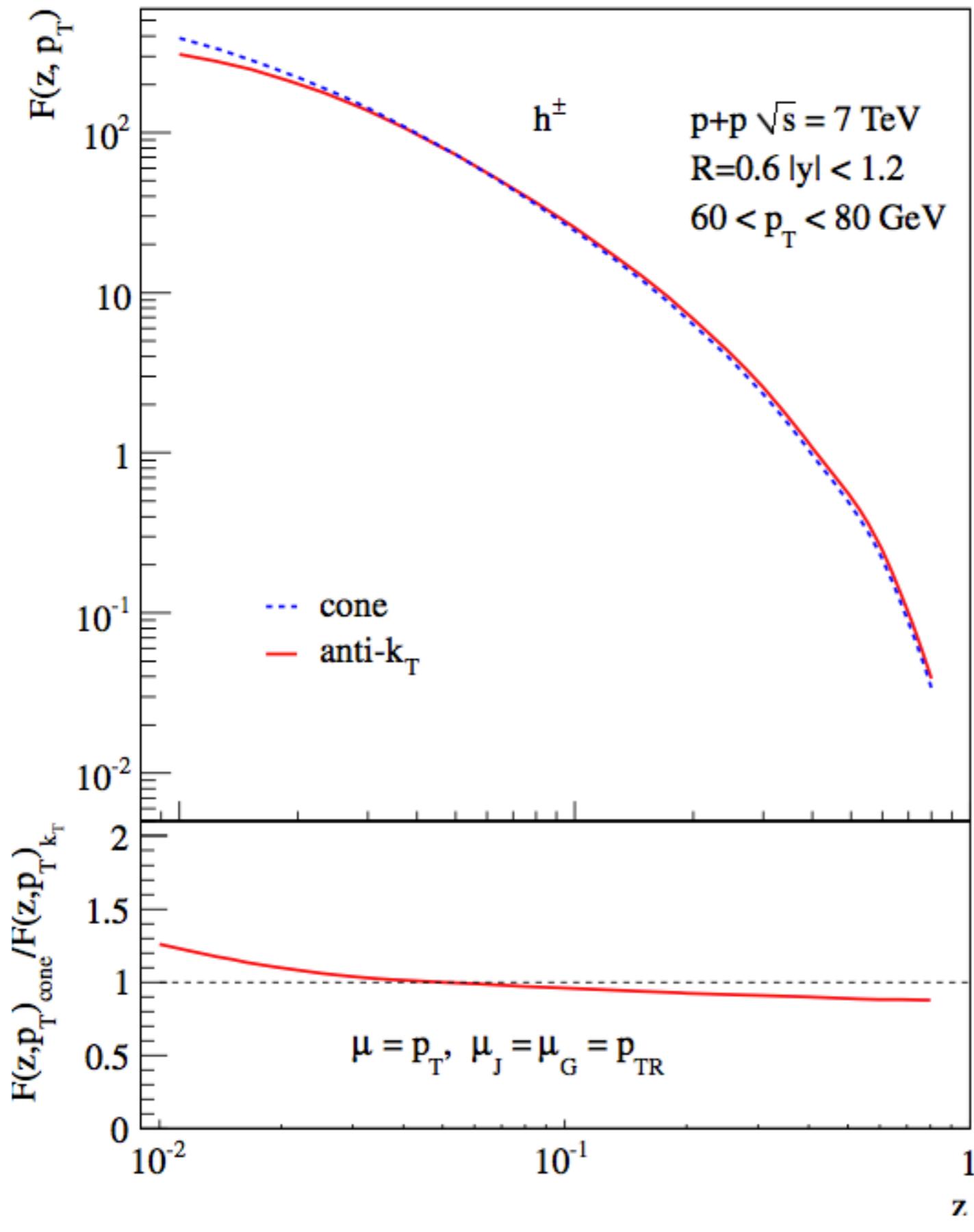
- Jet fragmentation function for light hadrons and heavy mesons
- Modification in heavy-ion collisions
- Threshold and small-z resummation
- Heavy quarks pp and AA
- e.g. hadron+jet back-to-back
- Extension to eA for the EIC



backup



Jet parameter R
dependence



Jet algorithm
dependence: cone, anti- k_T

Accuracy of Resummation

$$\mathcal{O}(\alpha_s^k) : \quad C_{kn} \times \alpha_s^k \ln^n \bar{N}, \quad \text{where } n \leq 2k$$

Fixed Order

LO	1						
NLO	$\alpha_s L^2$	$\alpha_s L$	α_s				
NNLO	$\alpha_s^2 L^4$	$\alpha_s^2 L^3$	$\alpha_s^2 L^2$	$\alpha_s^2 L$	α_s^2		
...	
$N^k LO$	$\alpha_s^k L^{2k}$	$\alpha_s^k L^{2k-1}$	$\alpha_s^k L^{2k-2}$	$\alpha_s^k L^{2k-3}$	$\alpha_s^k L^{2k-4}$...

↓ ↓ ↓

LL NLL NNLL

$$L = \ln \bar{N}$$

I. Initial state - Cold Nuclear Matter (CNM)

2. Final state - jet energy loss

Inclusive jet R_{AA} 